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UNBIASED L1 ESTIMATORS AND THEIR COVARIANCES,(U)
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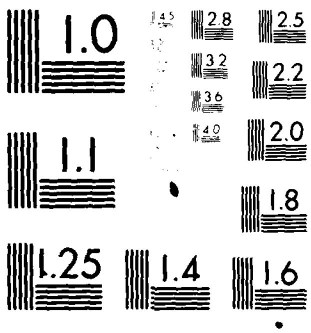

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Technical Report #64

UNBIASED L_1 ESTIMATORS AND THEIR COVARIANCES

by

Book, D., Booker, J.,
Hartley, H.O., and Sielken, R.L. Jr.

Texas A&M University
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

ATTACHMENT I

UNBIASED L_1 ESTIMATORS

AND THEIR COVARIANCES

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THEMIS OPTIMIZATION RESEARCH PROGRAM
Technical Report No. 64
June, 1980

INSTITUTE OF STATISTICS
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ATTACHMENT II

ABSTRACT

The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program for obtaining unbiased L_1 estimators and estimates of their covariances. These estimated covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 65 provides an analogous treatment of L_1 estimation subject to linear constraints on the parameters.

Table of Contents

Unbiased L_1 Estimators and their Covariances

1. An Introduction to MRS. A.

Consider the linear regression model in the form

$$y = X\beta + \varepsilon \tag{1}$$

where y is a vector of n observations, X is an n x p matrix of rank p of known constants, β is a vector of p unknown parameters and ε is a vector of independent random variables (noise) symmetrically distributed with mean zero and variance σ^2 . Unbiased estimation of β can be obtained under several different optimality criteria. The classical least squares approach is to estimate β by

$$\tilde{\beta} = (X^T X)^{-1} X^T Y \tag{2}$$

which has the smallest variance among the class of unbiased linear functions of y. The least squares estimator, $\tilde{\beta}$, is extremely sensitive to large values of $|\varepsilon|$, outliers, particularly when the sample size, n, is small relative to p, say $n \leq 2(p+1)$. This sensitivity suggests that an optimality criteria other than minimum variance should be considered. Several authors (Barrodale (1968), Charnes and Cooper (1964), Gentle, Kennedy and Sposito (1977), Harris (1950), Harter (1974), Rice and White (1964), Taylor (1973)) have suggested that

$$\sum_{i=1}^{n} | y_i - X_i\beta | \tag{3}$$

should be minimized with respect to β where y_i is the i-th observation and X_i is the i-th row of X. The estimator, $\hat{\beta}$, which minimizes the sum of the absolute residuals is often called the L_1 estimator.

Since the L_1 estimate is not necessarily unique, the unbiasedness of an L_1 estimator depends upon its method of computation. Hartley and Sielken [1973] have shown how to obtain an unbiased L_1 estimator using any conventional

linear programming algorithm and an initial unbiased antisymmetrical estimator of β, say β_0, where

$$\beta - \beta_0(\epsilon) = - [\beta - \beta_0(-\epsilon)] . \tag{4}$$

The computer program MRS. A implements an algorithm for Minimizing the Sum of the Absolute Residuals. The algorithm uses the least squares estimator $\tilde{\beta}$ as the initial unbiased antisymmetrical estimator.

It is nice to be able to compute an L_1 estimate. The fact that the L_1 estimator can be made unbiased is the first step in understanding its properties. The second step is to estimate its covariance. Such an estimate would usually be a prerequisite for confidence intervals or hypothesis tests. In the past the absence of such a covariance estimator has made L_1 estimation less attractive. MRS. A contains a mini-Monte Carlo procedure for estimating the covariance of the L_1 estimator. This feature sets MRS. A apart from other L_1 estimation procedures.

2. Computational Procedure

The problem of minimizing the sum of the absolute residuals can be formulated as follows:

$$\min \sum_{i=1}^{n} r_i \tag{5}$$

subject to

$$-r_i \leq y_i - X_i\beta \leq r_i \ , \ i = 1, \ldots, n \ , \tag{6}$$

$$r_i \geq 0 \ , \tag{7}$$

$$\beta \text{ unrestricted} \ , \tag{8}$$

where r_i is the i-th absolute residual. However, to insure that the resulting L_1 estimator is unbiased, the problem is reformulated following Hartley and Sielken [1973]. In particular, introducing the antisymmetrical least squares estimator, β_0, transforms (6) to

$$-r_i \leq y_i - X_i\beta_0 - X_i\,(\beta - \beta_0) \leq r_i \ , \ i = 1 \ , \ldots, n. \tag{9}$$

Then, using

$$\beta = \beta^{(1)} - \beta^{(2)} \ ,$$

$$\beta_0 = \beta_0^{(1)} - \beta_0^{(2)}$$

with $\beta^{(1)}, \beta^{(2)}, \beta_0^{(1)} , \beta_0^{(2)} \geq 0$ in (9) yields

$$-X_i\,(\beta^{(1)} + \beta_0^{(2)}) + X_i\,(\beta^{(2)} + \beta_0^{(1)}) - r_i \leq -\,y_i + X_i\beta_0$$

$$X_i\,(\beta^{(1)} + \beta_0^{(2)}) - X_i\,(\beta^{(2)} + \beta_0^{(1)}) - r_i \leq y_i - X_i\beta_0$$

or equivalently

$$-X_i B_1 + X_i B_2 - r_i \leq - y_i + X_i \beta_0$$

$$X_i B_1 - X_i B_2 - r_i \leq y_i - X_i \beta_0 \tag{10}$$

for $i = 1, \ldots, n$ where

$$B_1 = \beta^{(1)} + \beta_0^{(2)} \geq 0$$

$$B_2 = \beta^{(2)} + \beta_0^{(1)} \geq 0 \; . \tag{11}$$

Now, in order to compensate for any idiosyncrosies in the particular linear programming algorithm used to solve the problem in (5), (7), (10), (11), the problem is considered in two equivalent symmetrical forms, P_1 and P_2 , with MRS. A randomly selecting either P_1 or P_2 with probability $\frac{1}{2}$. The problems P_1 and P_2 in matrix notation are as follows:

P_1: $\min \sum_{i=1}^{n} r_i$

subject to

$$\begin{bmatrix} -X & X & -I \\ X & -X & -I \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ r \end{bmatrix} \leq \begin{bmatrix} -y + X\beta_0 \\ y - X\beta_0 \end{bmatrix} ,$$

$$B_1 , B_2 , r \geq 0 ;$$

P_2: $\min \sum_{i=1}^{n} r_i$

subject to

$$\begin{bmatrix} -X & X & -I \\ X & -X & -I \end{bmatrix} \begin{bmatrix} B_2 \\ B_1 \\ r \end{bmatrix} \leq \begin{bmatrix} y - X\beta_0 \\ -y + X\beta_0 \end{bmatrix} ,$$

$$B_1 \ , B_2 \ , r \geq 0.$$

After either P_1 or P_2 has been solved by MRS. A , the unbiased L_1 estimator is

$$\hat{\beta} = B_1 - B_2 + \beta_0 \ . \tag{12}$$

If the sample size was quite large, say $n >> (p + 1)^2$, the sample could be randomly subdivided into G groups, then $\hat{\beta}_g$ estimated for each group g separately, and the covariance of $\hat{\beta}$ estimated from the sample covariance of the $\hat{\beta}_g$'s.

MRS. A estimates the covariance of $\hat{\beta}$ using a mini-Monte Carlo procedure. Conceptually a Monte-Carlo estimate would be obtained by generating several sets of n y's , finding the L_1 estimate for each set, and computing a sample covariance. There are two difficulties with generating the y's; namely,

(1) β is unknown, and

(2) σ^2 is unknown.

The first of these difficulties can be overcome by expressing the sum of absolute residuals as

$$\sum_{i=1}^{n} | y_i - X_i\hat{\beta} |$$

$$= \sum_{i=1}^{n} | X_i\beta + \varepsilon_i - X_i\hat{\beta} |$$

$$= \sum_{i=1}^{n} | \varepsilon_i - X_i (\hat{\beta} - \beta) |$$

$$= \sum_{i=1}^{n} | \varepsilon_i - X_i \, \delta\beta | \tag{13}$$

using (1) and $\delta\beta = \hat{\beta} - \beta$ and noting that the covariance of the L_1 estimator of $\delta\beta$ is the same covariance of $\hat{\beta}$. Thus, only sets of n ε's need be generated. To deal with the second difficulty (the unknown σ^2), note that

$$\sum_{i=1}^{n} | \varepsilon_i - X_i \, \delta\beta |$$

$$= \sigma \sum_{i=1}^{n} | \varepsilon_i^* - X_i \, \delta\beta^* | \tag{14}$$

where $\delta\beta^* = (\hat{\beta} - \beta) / \sigma$ and ε_i / σ is symmetrically distributed with mean 0 and variance 1. Furthermore,

$$\text{Covariance } (\hat{\beta}) = \text{Covariance } (\hat{\delta\beta})$$
$$= \sigma^2 \text{Covariance } (\hat{\delta\beta}^*). \tag{15}$$

Thus, the mini-Monte Carlo procedure generates K sets of n ε^*'s; finds the K L_1 estimates $\hat{\delta\beta}^*$; and then estimates the covariance of $\hat{\beta}$ by $\hat{\sigma}$

times the sample covariance of the $\hat{\delta\beta}^*$.

Several possibilities for $\hat{\sigma}^2$ have been considered

$$(1)\quad \hat{\sigma}_1^2 = \sum_{i=1}^{n} (y_i - X_i\beta_0)^2 \,/\, (n-p),$$

$$(2)\quad \hat{\sigma}_2^2 = \sum_{i=1}^{n} (y_i - X_i\hat{\beta})^2 \,/\, (n-p),$$

$$(3)\quad \hat{\sigma}_3^2 = \frac{\sum_{i=1}^{n} (y_i - X_i\hat{\beta})^2 \,/\, (n-p)}{\frac{1}{K}\sum_{k=1}^{K} (\varepsilon_{ik}^* - X_i\,\hat{\delta\beta}_k^*)^2 \,/\, (n-p)},$$

$$(4)\quad \hat{\sigma}_4^2 = \left\{\frac{\sum_{i=1}^{n} |\, y_i - X_i\hat{\beta}\,| \,/\, (n-p)}{\frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{n} |\,\varepsilon_{ik}^* - X_i\,\hat{\delta\beta}_k^*\,| \,/\, (n-p)}\right\}^2 .$$

Of course, $\hat{\sigma}_1^2$ is the usual least squares estimator of σ^2 while $\hat{\sigma}_2^2$ has the same form but uses the L_1 estimator of β instead of the least squares estimator. The ratio estimators, $\hat{\sigma}_3^2$ and $\hat{\sigma}_4^2$ reflect the fact that the variance of the y's is σ^2 while the variance of the ε^*'s is 1 . Since L_1 estimation is used to avoid certain weaknesses in least squares estimation, it seems appropriate to estimate σ^2 using the absolute residuals, as in $\hat{\sigma}_4^2$, instead of the squared residuals. Furthermore $E\,[|y - X\beta|]$ is often proportional to σ. In particular,

$$E\,[|y - X\beta|] = \sigma\,(2/\sqrt{3})^{-1}\,, \qquad (16)$$

if ε has a uniform distribution;

$$E\,[\,|\,y - X\beta\,|\,] = \sigma\,(\sqrt{\pi/2})^{-1}\,, \qquad (17)$$

if ε has a normal distribution; and

$$E\,[\,|\,y - X\beta\,|\,] = \sigma\,(\sqrt{2})^{-1}\,, \qquad (18)$$

if ε has a double exponential distribution. Of course, if

$$E\,[\,|\,y - X\beta\,|\,] = C\sigma \qquad (19)$$

and the ε^*'s are generated from the same distribution but with variance 1, then the proportionality constant doesn't affect $\hat{\sigma}_4^2$. For these reasons MRS. A uses $\hat{\sigma}_4^2$.

The results in (16), (17), and (18) suggest another alternative for $\hat{\sigma}$; namely,

$$\hat{\sigma}_5 = \frac{1}{C} \sum_{i=1}^{n} |\, y_i - X_i\hat{\beta}\,| \,/\, (n-p)$$

where C depends on the assumed distribution of ε . Note that $\hat{\sigma}_5$ reflects only the variability in $Y_i - X_i\beta$ whereas $\hat{\sigma}_4$ reflects not only this variability but also the variability associated with the linear programming algorithm. Some empirical behavior of these five estimators of σ is reported in Table 1.

MRS. A allows the user to generate the ε^*'s from either the uniform, normal, or double exponential distributions. These distributions were selected as being representative of short, medium, and long tailed distributions respectively. These three distributions are also interesting because maximum likelihood corresponds to minimizing the maximum absolute

Table 1

Empirical Results on Five Estimators of σ^2

For n = 20

σ^2	$\hat{\sigma}_1^2$	$\hat{\sigma}_2^2$	$\hat{\sigma}_3^2$	$\hat{\sigma}_4^2$	$\hat{\sigma}_5^2$	ε distribution
25	21.14	21.66	20.36	24.24	23.45	Uniform
25	21.59	21.66	20.26	22.58	23.61	Normal
25	40.53	45.47	48.77	36.22	36.50	Double Exponential
100	111.63	119.44	109.54	110.12	113.59	Uniform
100	87.64	87.92	81.21	91.21	95.67	Normal
100	38.25	40.06	34.43	39.20	45.93	Double Exponential
400	375.31	376.33	351.39	360.95	362.25	Uniform
400	349.55	350.70	324.38	363.56	381.67	Normal
400	333.70	336.78	300.91	214.12	251.63	Double Exponential
2500	3255.33	3782.59	3551.58	3762.09	3724.80	Uniform
2500	2174.03	2181.34	2029.00	2270.24	2378.90	Normal
2500	2480.54	2907.44	2497.52	2345.71	2720.68	Double Exponential
8100	8363.82	9644.51	9027.74	8310.63	8470.63	Uniform
8100	7117.80	7139.82	6681.97	7433.47	7765.47	Normal
8100	7987.26	8063.69	7608.09	8355.12	9082.50	Double Exponential

residual if the ε's are uniform, minimizing the sum of squared residuals if the ε's are normal, and minimizing the sum of absolute residuals if ε's are double exponential.

Since the proportionality constants $2/\sqrt{3}$, $\sqrt{\pi/2}$, and $\sqrt{2}$ or approximately 1.155, 1.253, 1.414 respectively are all nearly the same, the estimator $\hat{\sigma}_4$ is not too sensitive to the possibility that $Y - X\beta$ and ε^* have different distributional forms.

MRS. A allows the option of assigning weights to the residuals modifying equation (5) to

$$\min \sum_{i=1}^{n} W_i r_i$$

where

$$W_i = \text{the weight given to the i-th residual.}$$

If W_i are not all equal to one then the objective function in the mini-Monte Carlo study is

$$\sum_{i=1}^{n} W_i | \varepsilon_i^* - X_i \delta\beta^* |$$

and the estimates of σ also reflect the W_i ; namely,

$$\hat{\sigma}_4 = \frac{\sum_{i=1}^{n} W_i |y_i - X_i \hat{\beta}|/(n-p)}{\frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{n} W_i |\varepsilon_{ik}^* - X_i \delta\hat{\beta}_k^* |/(n-p)}$$

and

$$\hat{\sigma}_5 = \frac{1}{C} \sum_{i=1}^{n} W_i |y_i - X_i \hat{\beta}|/(n-p) \quad .$$

3. MRS. A: User's Guide and Sample Problem

MRS. A consists of a main program and seven subroutines. The functions of these components are described in Table 2.

Input instructions are briefly documented in the program and consist of four basic card types.

The first card or card image contains the ancillary statistics for the specific problem as follows:

First Card:

Card Column	Variable Name	Description
1-5	NOBS	= n = number of observations (format I5; i.e., a 5 digit integer, right justified)
6-10	IP	= p = number of beta parameters (format I5)
11-15	ISAM	= K = number of samples for the mini-Monte Carlo study (format I5)
16-26	NSEED	Ten digit random number $\leq$ 2147483647 (format I11)
28	IWRIT1	= 1 if the main results of L_1 estimation are printed = 0 otherwise
30	IWRIT2	= 1 if the intermediate results of L_1 estimation are printed = 0 otherwise
32	IWRIT3	= 1 if the main results of the mini-Monte Carlo study are printed = 0 otherwise

Card Column	Variable Name	Description
34	IWRIT4	= 1 if the intermediate results of the mini-Monte Carlo study are printed = 0 otherwise
36	IWRIT5	= 1 if the inputted data are printed = 0 otherwise
38	IWRIT6	= 1 if the intermediate steps in the determination of the covariance of $\hat{\beta}$ are printed = 0 otherwise
40	IOPTN	= 1 if the ε^*'s are to be normally distributed = 2 if the ε^*'s are to be double exponentially distributed = 3 if the ε^*'s are to be uniformly distributed
42	IWT	= 1 if weights, W_i , are to be assigned to the residuals = 0 if residuals are not weighted

The remaining card input instructions are as follows:

Card Number	Variable Name	Description
Second card group:	W_i, i=1, ..., NOBS	The weights assigned to the residuals [format (8F10.5) ; i.e., eight ten digit numbers with either a decimal point included or last 5 digits are assumed to be to the right of a supplied decimal point.]

Card Number	Variable Name	Description
Third card group:	y_i, i=1, ..., NOBS	The observations [format(8F10.5)]
Fourth card group:	X_{ij}, i=1, ..., NOBS j=1, ..., IP	The matrix of beta coefficients read in by rows [format (8F10.5)]

The user may also supply a title card of 80 spaces or less following the fourth card group.

The size of the problem which can be solved is limited only by the dimension statements in MRS. A. Currently these restrict the size to be 20 or less observations ($n \leq 20$), 10 or less parameters ($p \leq 10$) and 100 or less samples in the mini-Monte Carlo study ($K \leq 100$). However, expansion can easily be accomplished by increasing these dimensions in the dimension statements as documented in the program.

MRS. A is written in Fortran IV language and is compatible with Fortran G and H and WATFIV language compilers. The program uses double precision arithmatic.

MRS. A has been tested on several problems on an AMDAHL 470 V6 and should be compatible with all IBM computers. MRS. A executes small problems such as $n = 5$, $p = 3$, $K = 6$ in less than two seconds. Problems of sizes $n = 20$, $p = 2$, $K = 30$ take up to a minute of execution time.

The sample input and sample output for a sample problem are given in Appendices A and B, respectively. The program listing is given in Appendix C.

Table 2

Components of MRS. A and Their Functions

Component	Function
MAIN	Reads data and generates output.
	Performs L_1 estimation.
	Carries out the Mini-Monte Carlo study.
	Determines $\hat{\sigma}$ and the estimated covariance of $\hat{\beta}$.
INVERT	Inverts an n x n matrix.
CONST	Constructs the least squares estimate of β, β_0.
XTXINV	Calculates $(X^TX)^{-1}$ for use in forming β_0 .
RAND	Generates random uniform variable with range 0 to 1.
NORMAL	Generates a vector of normally distributed ε^* with mean 0 and variance 1 for use in the mini-Monte Carlo study.
DOUBLE	Generates a vector of double exponentially distributed ε^* with mean 0 and variance 1 for use in the mini-Monte Carlo study.
UNIFORM	Generates a vector of uniformly distributed ε^* with mean 0 and variance 1 for use in the mini-Monte Carlo study.

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APPENDIX A. SAMPLE INPUTS

```
 5    2    20 1872539680 0 0 0 0 0 0 1 0
4.25700  -1.98300   0.02400  -3.18000  -3.98600
1.00000   1.12000
1.00000   1.95000
1.00000   3.02000
1.00000   5.43000
1.00000   6.59000
THIS IS AN EXAMPLE PROBLEM OF MRS. A. WITH NO OPTIONAL PRINTOUTS.
```

```
 5     2    20 1872539680 1 1 1 1 1 1 1 0
4.25700  -1.98300   0.02400  -3.18000  -3.98600
1.00000   1.12000
1.00000   1.95000
1.00000   3.02000
1.00000   5.43000
1.00000   6.59000
THIS IS AN EXAMPLE PROBLEM OF MRS. A. WITH ALL OPTIONAL PRINTOUTS.
```

APPENDIX B. SAMPLE OUTPUTS

```
          MRS. A :

MINIMIZES SUM OF ABSOLUTE RESIDUALS.

     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN
ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED
REGRESSION PARAMETERS.
     UNBIASED ESTIMATES OF THE REGRESSION
PARAMETERS ARE OBTAINED USING THE PROCEDURE
DESCRIBED IN A PAPER BY H.O. HARTLEY AND R.L. SIELKEN,JR,
'TWO LINEAR PROGRAMMING ALGORITHMS FOR UNBIASED ESTIMATION
OF LINEAR MODELS', 1973, JASA, VOL. 68, PAGES 639-41.

     THE FOLLOWING PROCEDURE DEVELOPED BY :
         D.N. BOOK
         J.B. BOOKER
         H.O. HARTLEY
         R.L. SIELKEN, JR.
     INSTITUTE OF STATISTICS
     TEXAS A & M UNIVERSTIY
     COLLEGE STATION, TEXAS 77843

     INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:
         ROBERT L. SIELKEN, JR.

     THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED.

   THIS IS AN EXAMPLE PROBLEM OF MRS. A. WITH NO OPTIONAL PRINTOUTS.

NUMBER OF OBSERVATIONS =      5
NUMBER OF PARAMETERS =     2
THE SAMPLE SIZE FOR THE MINI-MONTE CARLO STUDY =    20
USER SUPPLIED INITIAL RANDOM INTEGER: NSEED = 1872539680
THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0, OF THE REGRESSION PARAMETER VECTOR, BETA
       LEAST SQUARES ESTIMATE OF BETA( 1) =       3.227002
       LEAST SQUARES ESTIMATE OF BETA( 2) =      -1.159747
MRS. A'S ANSWER :    THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS:
       L1 ESTIMATE OF BETA( 1) =       3.416213
       L1 ESTIMATE OF BETA( 2) =      -1.123249
THE RESIDUALS, R(I),I=1,NOBS
                2.098826
                3.208877
                0.000000
                0.496969
                0.000000
THE SUM OF THE ABSOLUTE RESIDUALS =       5.804672
THE MAXIMUM ABSOLUTE RESIDUAL =       3.208877
MAIN RESULTS OF THE MINI-MONTE CARLO STUDY
ESTIMATED VALUE OF SIGMA (SIGMA HAT 4) =       2.392353
ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR (BETA) USING THIS ESTIMATE OF SIGMA
        4.433552     -0.849803
       -0.849803      0.364199
```

(Sample output with no optional printouts)

```
             MRS. A :

MINIMIZES SUM OF ABSOLUTE RESIDUALS.
```

(Sample output with optional printouts)

```
     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN
ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED
REGRESSION PARAMETERS.
     UNBIASED ESTIMATES OF THE REGRESSION
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OF LINEAR MODELS', 1973, JASA, VOL. 68, PAGES 639-41.

     THE FOLLOWING PROCEDURE DEVELOPED BY :
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   THIS IS AN EXAMPLE PROBLEM OF MRS. A. WITH ALL OPTIONAL PRINTOUTS.

NUMBER OF OBSERVATIONS =     5
NUMBER OF PARAMETERS =     2
THE SAMPLE SIZE FOR THE MINI-MONTE CARLO STUDY =    20
USER SUPPLIED INITIAL RANDOM INTEGER: NSEED =  1872539680
THE LINEAR REGRESSION IS Y = X*BETA + EPSILON
WHERE
Y CONTAINS THE OBSERVATIONS,
BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS,
THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA
CORRESPONDING TO THE I-TH OBSERVATION, AND
EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE
SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM
X*BETA.
     THE Y VECTOR
                  4.25700
                 -1.98300
                  0.02400
                 -3.18000
                 -3.98600
     THE X MATRIX
              1.0    1.1
              1.0    2.0
              1.0    3.0
              1.0    5.4
              1.0    6.6
SUPPLEMENTAL INFORMATION FROM THE LINEAR PROGRAMMING PROBLEM DETERMINATION OF THE ESTIMATE OF
THE REGRESSION PARAMETER VECTOR, BETA.
     THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED
          THE OBJECTIVE FUNCTION COEFFICIENTS
               C(  1) =       0.00000
               C(  2) =       0.00000
               C(  3) =       0.00000
               C(  4) =       0.00000
               C(  5) =      -1.00000
               C(  6) =      -1.00000
               C(  7) =      -1.00000
               C(  8) =      -1.00000
               C(  9) =      -1.00000
               C( 10) =       0.00000
               C( 11) =       0.00000
               C( 12) =       0.00000
               C( 13) =       0.00000
               C( 14) =       0.00000
               C( 15) =       0.00000
               C( 16) =       0.00000
               C( 17) =       0.00000
               C( 18) =       0.00000
               C( 19) =       0.00000
     THE CONSTRAINT MATRIX A
  -1.0   -1.1    1.0    1.1   -1.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0    0.0    0.0    0.0    0.0
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0.0    0.0    0.0
    -1.0   -2.0    1.0    2.0    0.0   -1.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0
    -1.0   -3.0    1.0    3.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0
    -1.0   -5.4    1.0    5.4    0.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0    0.0
0.0    0.0    0.0
    -1.0   -6.6    1.0    6.6    0.0    0.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    1.0    0.0    0.0
0.0    0.0    0.0
     1.0    1.1   -1.0   -1.1   -1.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1.0    0.0
0.0    0.0    0.0
     1.0    2.0   -1.0   -2.0    0.0   -1.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1.0
0.0    0.0    0.0
     1.0    3.0   -1.0   -3.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
1.0    0.0    0.0
     1.0    5.4   -1.0   -5.4    0.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    1.0    0.0
     1.0    6.6   -1.0   -6.6    0.0    0.0    0.0    0.0   -1.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    1.0
    THE VARIABLES INITIALLY IN THE BASIS
         10,11,12,13,14,15,16,17,18,19,
    THE PROBLEM P2 HAS BEEN SELECTED
    THE INVERSE OF XTX
         0.810298     -0.168497
        -0.168497      0.046521
    VALUE OF BETA0 TO COMPUTE RHS
         3.227002     -1.159747
    THE RHS, YMXB, FOR P1 OR P2
          2.328914
         -2.948496
          0.299433
         -0.109578
          0.429728
    THE INITIAL VALUES OF THE BASIC VARIABLES
         XB(  1) =     0.23289D 01
         XB(  2) =    -0.29485D 01
         XB(  3) =     0.29943D 00
         XB(  4) =    -0.10958D 00
         XB(  5) =     0.42973D 00
         XB(  6) =    -0.23289D 01
         XB(  7) =     0.29485D 01
         XB(  8) =    -0.29943D 00
         XB(  9) =     0.10958D 00
         XB( 10) =    -0.42973D 00
    THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =     0.00000D 00
    THE REDUCED COSTS
         THE   1-TH REDUCED COST =    -0.00000D 00
         THE   2-TH REDUCED COST =    -0.00000D 00
         THE   3-TH REDUCED COST =    -0.00000D 00
         THE   4-TH REDUCED COST =    -0.00000D 00
         THE   5-TH REDUCED COST =    -0.10000D 01
         THE   6-TH REDUCED COST =    -0.10000D 01
         THE   7-TH REDUCED COST =    -0.10000D 01
         THE   8-TH REDUCED COST =    -0.10000D 01
         THE   9-TH REDUCED COST =    -0.10000D 01
         THE  10-TH REDUCED COST =    -0.00000D 00
         THE  11-TH REDUCED COST =    -0.00000D 00
         THE  12-TH REDUCED COST =    -0.00000D 00
         THE  13-TH REDUCED COST =    -0.00000D 00
         THE  14-TH REDUCED COST =    -0.00000D 00
         THE  15-TH REDUCED COST =    -0.00000D 00
         THE  16-TH REDUCED COST =    -0.00000D 00
         THE  17-TH REDUCED COST =    -0.00000D 00
         THE  18-TH REDUCED COST =    -0.00000D 00
         THE  19-TH REDUCED COST =    -0.00000D 00
    TENTATIVELY THE   2-TH BASIC VARIABLE IS LEAVING THE BASIS
    THE YRJ'S
    (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
    IT CAN BE ZERO OTHERWISE TOO.)
         YR(  1) =    -0.10000D 01
         YR(  2) =    -0.19500D 01
         YR(  3) =     0.10000D 01
         YR(  4) =     0.19500D 01
         YR(  5) =     0.00000D 00
         YR(  6) =    -0.10000D 01
         YR(  7) =     0.00000D 00
         YR(  8) =     0.00000D 00
         YR(  9) =     0.00000D 00
         YR( 10) =     0.00000D 00
         YR( 11) =     0.00000D 00
         YR( 12) =     0.00000D 00
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     YR( 13) =      0.00000D 00
     YR( 14) =      0.00000D 00
     YR( 15) =      0.00000D 00
     YR( 16) =      0.00000D 00
     YR( 17) =      0.00000D 00
     YR( 18) =      0.00000D 00
     YR( 19) =      0.00000D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   1-TH NET PRICE TO YR(  1) =     -0.00000D 00
     THE RATIO OF THE   2-TH NET PRICE TO YR(  2) =     -0.00000D 00
     THE RATIO OF THE   6-TH NET PRICE TO YR(  6) =     -0.10000D 01
THE   2-TH VARIABLE IS LEAVING THE BASIS.
THE   2-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
     10,   2,  12,  13,  14,  15,  16,  17,  18,  19,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.40224D 01
     XB(  2) =      0.15120D 01
     XB(  3) =      0.48658D 01
     XB(  4) =      0.81009D 01
     XB(  5) =      0.10394D 02
     XB(  6) =     -0.40224D 01
     XB(  7) =      0.22204D-15
     XB(  8) =     -0.48658D 01
     XB(  9) =     -0.81009D 01
     XB( 10) =     -0.10394D 02
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.00000D 00
THE REDUCED COSTS
     THE   1-TH REDUCED COST =     -0.00000D 00
     THE   2-TH REDUCED COST =     -0.00000D 00
     THE   3-TH REDUCED COST =     -0.00000D 00
     THE   4-TH REDUCED COST =     -0.00000D 00
     THE   5-TH REDUCED COST =     -0.10000D 01
     THE   6-TH REDUCED COST =     -0.10000D 01
     THE   7-TH REDUCED COST =     -0.10000D 01
     THE   8-TH REDUCED COST =     -0.10000D 01
     THE   9-TH REDUCED COST =     -0.10000D 01
     THE  10-TH REDUCED COST =     -0.00000D 00
     THE  11-TH REDUCED COST =     -0.00000D 00
     THE  12-TH REDUCED COST =     -0.00000D 00
     THE  13-TH REDUCED COST =     -0.00000D 00
     THE  14-TH REDUCED COST =     -0.00000D 00
     THE  15-TH REDUCED COST =     -0.00000D 00
     THE  16-TH REDUCED COST =     -0.00000D 00
     THE  17-TH REDUCED COST =     -0.00000D 00
     THE  18-TH REDUCED COST =     -0.00000D 00
     THE  19-TH REDUCED COST =     -0.00000D 00
TENTATIVELY THE  10-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     -0.23795D 01
     YR(  2) =      0.00000D 00
     YR(  3) =      0.23795D 01
     YR(  4) =     -0.44409D-15
     YR(  5) =      0.00000D 00
     YR(  6) =     -0.33795D 01
     YR(  7) =      0.00000D 00
     YR(  8) =      0.00000D 00
     YR(  9) =     -0.10000D 01
     YR( 10) =      0.00000D 00
     YR( 11) =      0.33795D 01
     YR( 12) =      0.00000D 00
     YR( 13) =      0.00000D 00
     YR( 14) =      0.00000D 00
     YR( 15) =      0.00000D 00
     YR( 16) =      0.00000D 00
     YR( 17) =      0.00000D 00
     YR( 18) =      0.00000D 00
     YR( 19) =      0.00000D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   1-TH NET PRICE TO YR(  1) =     -0.00000D 00
     THE RATIO OF THE   6-TH NET PRICE TO YR(  6) =     -0.29590D 00
     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =     -0.10000D 01
THE  10-TH VARIABLE IS LEAVING THE BASIS.
THE   1-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
     10,   2,  12,  13,  14,  15,  16,  17,  18,   1,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.58817D 01
     XB(  2) =     -0.72807D 00
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     XB(  3) =      0.24689D 01
     XB(  4) =      0.30525D 00
     XB(  5) =      0.66613D-15
     XB(  6) =     -0.58817D 01
     XB(  7) =      0.10080D-15
     XB(  8) =     -0.24689D 01
     XB(  9) =     -0.30525D 00
     XB( 10) =      0.43682D 01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.00000D 00
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =    -0.00000D 00
     THE   5-TH REDUCED COST =    -0.10000D 01
     THE   6-TH REDUCED COST =    -0.10000D 01
     THE   7-TH REDUCED COST =    -0.10000D 01
     THE   8-TH REDUCED COST =    -0.10000D 01
     THE   9-TH REDUCED COST =    -0.10000D 01
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.00000D 00
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.00000D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.00000D 00
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.00000D 00
TENTATIVELY THE   6-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =      0.00000D 00
     YR(  2) =      0.00000D 00
     YR(  3) =     -0.18041D-15
     YR(  4) =     -0.44409D-15
     YR(  5) =     -0.10000D 01
     YR(  6) =     -0.11789D 01
     YR(  7) =      0.00000D 00
     YR(  8) =      0.00000D 00
     YR(  9) =     -0.17888D 00
     YR( 10) =      0.00000D 00
     YR( 11) =      0.11789D 01
     YR( 12) =      0.00000D 00
     YR( 13) =      0.00000D 00
     YR( 14) =      0.00000D 00
     YR( 15) =      0.00000D 00
     YR( 16) =      0.00000D 00
     YR( 17) =      0.00000D 00
     YR( 18) =      0.00000D 00
     YR( 19) =      0.17888D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   5-TH NET PRICE TO YR(  5) =    -0.10000D 01
     THE RATIO OF THE   6-TH NET PRICE TO YR(  6) =    -0.84826D 00
     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =    -0.55904D 01
THE   6-TH VARIABLE IS LEAVING THE BASIS.
THE   6-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
      10,   2,  12,  13,  14,   6,  16,  17,  18,   1,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.22204D-15
     XB(  2) =      0.34720D 00
     XB(  3) =     -0.13698D 01
     XB(  4) =     -0.94206D 00
     XB(  5) =     -0.15495D-14
     XB(  6) =      0.49892D 01
     XB(  7) =      0.99785D 01
     XB(  8) =      0.13698D 01
     XB(  9) =      0.94206D 00
     XB( 10) =     -0.27178D 01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.49892D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =     0.13878D-15
     THE   4-TH REDUCED COST =     0.27756D-15
     THE   5-TH REDUCED COST =    -0.15174D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.10000D 01
     THE   8-TH REDUCED COST =    -0.10000D 01
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     THE   9-TH REDUCED COST =    -0.84826D 00
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.00000D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.84826D 00
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.15174D 00
TENTATIVELY THE  10-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =    -0.10000D 01
     YR(  4) =     0.00000D 00
     YR(  5) =    -0.12048D 01
     YR(  6) =     0.00000D 00
     YR(  7) =     0.00000D 00
     YR(  8) =     0.00000D 00
     YR(  9) =     0.20475D 00
     YR( 10) =     0.00000D 00
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
     YR( 13) =     0.00000D 00
     YR( 14) =     0.00000D 00
     YR( 15) =     0.12048D 01
     YR( 16) =     0.00000D 00
     YR( 17) =     0.00000D 00
     YR( 18) =     0.00000D 00
     YR( 19) =    -0.20475D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   3-TH NET PRICE TO YR(  3) =     0.13878D-15
     THE RATIO OF THE   5-TH NET PRICE TO YR(  5) =    -0.12595D 00
     THE RATIO OF THE  19-TH NET PRICE TO YR( 19) =    -0.74107D 00
THE  10-TH VARIABLE IS LEAVING THE BASIS.
THE   3-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
    10,   2,  12,  13,  14,   6,  16,  17,  18,   3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =     0.14661D-15
     XB(  2) =     0.34720D 00
     XB(  3) =    -0.13698D 01
     XB(  4) =    -0.94206D 00
     XB(  5) =    -0.53118D-15
     XB(  6) =     0.49892D 01
     XB(  7) =     0.99785D 01
     XB(  8) =     0.13698D 01
     XB(  9) =     0.94206D 00
     XB( 10) =     0.27178D 01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.49892D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =     0.26368D-15
     THE   5-TH REDUCED COST =    -0.15174D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.10000D 01
     THE   8-TH REDUCED COST =    -0.10000D 01
     THE   9-TH REDUCED COST =    -0.84826D 00
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.00000D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.84826D 00
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.15174D 00
TENTATIVELY THE   3-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =    -0.13878D-16
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
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     YR( 4) =     0.00000D 00
     YR( 5) =    -0.65265D 00
     YR( 6) =     0.00000D 00
     YR( 7) =    -0.10000D 01
     YR( 8) =     0.00000D 00
     YR( 9) =    -0.34735D 00
     YR( 10) =    0.00000D 00
     YR( 11) =    0.00000D 00
     YR( 12) =    0.00000D 00
     YR( 13) =    0.00000D 00
     YR( 14) =    0.00000D 00
     YR( 15) =    0.65265D 00
     YR( 16) =    0.00000D 00
     YR( 17) =    0.00000D 00
     YR( 18) =    0.00000D 00
     YR( 19) =    0.34735D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   5-TH NET PRICE TO YR(  5) =    -0.23249D 00
     THE RATIO OF THE   7-TH NET PRICE TO YR(  7) =    -0.10000D 01
     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =    -0.24421D 01
THE   3-TH VARIABLE IS LEAVING THE BASIS.
THE   5-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
      10,   2,   5,  13,  14,   6,  16,  17,  18,   3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =    0.41977D 01
     XB(  2) =   -0.36497D-01
     XB(  3) =    0.20988D 01
     XB(  4) =   -0.49697D 00
     XB(  5) =   -0.68800D-15
     XB(  6) =    0.32089D 01
     XB(  7) =    0.64178D 01
     XB(  8) =    0.22204D-15
     XB(  9) =    0.49697D 00
     XB( 10) =    0.18921D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.53077D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.20817D-15
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =     0.22204D-15
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.76751D 00
     THE   8-TH REDUCED COST =    -0.10000D 01
     THE   9-TH REDUCED COST =    -0.76751D 00
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.23249D 00
     THE  13-TH REDUCED COST =    -0.00000D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.23249D 00
TENTATIVELY THE   4-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =   -0.13878D-16
     YR(  2) =    0.00000D 00
     YR(  3) =    0.00000D 00
     YR(  4) =   -0.66613D-15
     YR(  5) =    0.00000D 00
     YR(  6) =    0.00000D 00
     YR(  7) =    0.32493D 00
     YR(  8) =   -0.10000D 01
     YR(  9) =   -0.67507D 00
     YR( 10) =    0.00000D 00
     YR( 11) =    0.00000D 00
     YR( 12) =   -0.32493D 00
     YR( 13) =    0.00000D 00
     YR( 14) =    0.00000D 00
     YR( 15) =    0.00000D 00
     YR( 16) =    0.00000D 00
     YR( 17) =    0.00000D 00
     YR( 18) =    0.00000D 00
     YR( 19) =    0.67507D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   8-TH NET PRICE TO YR(  8) =    -0.10000D 01
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     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =     -0.11369D 01
     THE RATIO OF THE  12-TH NET PRICE TO YR( 12) =     -0.71552D 00
THE   4-TH VARIABLE IS LEAVING THE BASIS.
THE  12-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
      10,   2,   5,  12,  14,   6,  16,  17,  18,   3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.88846D 01
     XB(  2) =     -0.46492D 00
     XB(  3) =      0.44423D 01
     XB(  4) =      0.15295D 01
     XB(  5) =     -0.86309D-15
     XB(  6) =      0.12210D 01
     XB(  7) =      0.24420D 01
     XB(  8) =     -0.15295D 01
     XB(  9) =      0.55511D-16
     XB( 10) =     -0.26341D 01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.56633D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.12490D-15
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =     0.66613D-15
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.10000D 01
     THE   8-TH REDUCED COST =    -0.28448D 00
     THE   9-TH REDUCED COST =    -0.28448D 00
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.71552D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.71552D 00
TENTATIVELY THE  10-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =    -0.10000D 01
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
     YR(  4) =    -0.35527D-14
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =     0.00000D 00
     YR(  8) =    -0.56810D 01
     YR(  9) =    -0.46810D 01
     YR( 10) =     0.00000D 00
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
     YR( 13) =     0.56810D 01
     YR( 14) =     0.00000D 00
     YR( 15) =     0.00000D 00
     YR( 16) =     0.00000D 00
     YR( 17) =     0.00000D 00
     YR( 18) =     0.00000D 00
     YR( 19) =     0.46810D 01
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   1-TH NET PRICE TO YR(  1) =     -0.12490D-15
     THE RATIO OF THE   8-TH NET PRICE TO YR(  8) =     -0.50076D-01
     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =     -0.60773D-01
THE  10-TH VARIABLE IS LEAVING THE BASIS.
THE   1-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
      10,   2,   5,  12,  14,   6,  16,  17,  18,   1,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.88846D 01
     XB(  2) =     -0.46492D 00
     XB(  3) =      0.44423D 01
     XB(  4) =      0.15295D 01
     XB(  5) =     -0.86309D-15
     XB(  6) =      0.12210D 01
     XB(  7) =      0.24420D 01
     XB(  8) =     -0.15295D 01
     XB(  9) =      0.18956D-16
     XB( 10) =      0.26341D 01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.56633D 01
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THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =     0.41633D-16
     THE   4-TH REDUCED COST =     0.66613D-15
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.10000D 01
     THE   8-TH REDUCED COST =    -0.28448D 00
     THE   9-TH REDUCED COST =    -0.28448D 00
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.71552D 00
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.00000D 00
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.71552D 00
TENTATIVELY THE   8-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
     YR(  4) =    -0.15543D-14
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =    -0.10000D 01
     YR(  8) =    -0.30776D 01
     YR(  9) =    -0.20776D 01
     YR( 10) =     0.00000D 00
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
     YR( 13) =     0.30776D 01
     YR( 14) =     0.00000D 00
     YR( 15) =     0.00000D 00
     YR( 16) =     0.00000D 00
     YR( 17) =     0.00000D 00
     YR( 18) =     0.00000D 00
     YR( 19) =     0.20776D 01
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   7-TH NET PRICE TO YR(  7) =    -0.10000D 01
     THE RATIO OF THE   8-TH NET PRICE TO YR(  8) =    -0.92437D-01
     THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =    -0.13693D 00
THE   8-TH VARIABLE IS LEAVING THE BASIS.
THE   8-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
     10,   2,   5,  12,  14,   6,  16,   8,  18,   1,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =     0.41977D 01
     XB(  2) =    -0.36497D-01
     XB(  3) =     0.20988D 01
     XB(  4) =     0.44409D-15
     XB(  5) =    -0.68800D-15
     XB(  6) =     0.32089D 01
     XB(  7) =     0.64178D 01
     XB(  8) =     0.49697D 00
     XB(  9) =     0.99394D 00
     XB( 10) =    -0.18921D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.58047D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.12490D-15
     THE   4-TH REDUCED COST =     0.66613D-15
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.90756D 00
     THE   8-TH REDUCED COST =    -0.00000D 00
     THE   9-TH REDUCED COST =    -0.92437D-01
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.10000D 01
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.92437D-01
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     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.90756D 00
TENTATIVELY THE  10-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =    -0.10000D 01
     YR(  4) =     0.46629D-14
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =    -0.18459D 01
     YR(  8) =     0.00000D 00
     YR(  9) =     0.84594D 00
     YR( 10) =     0.00000D 00
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
     YR( 13) =     0.00000D 00
     YR( 14) =     0.00000D 00
     YR( 15) =     0.00000D 00
     YR( 16) =     0.00000D 00
     YR( 17) =     0.18459D 01
     YR( 18) =     0.00000D 00
     YR( 19) =    -0.84594D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   3-TH NET PRICE TO YR(  3) =    -0.12490D-15
     THE RATIO OF THE   7-TH NET PRICE TO YR(  7) =    -0.49165D 00
     THE RATIO OF THE  19-TH NET PRICE TO YR( 19) =    -0.10728D 01
THE  10-TH VARIABLE IS LEAVING THE BASIS.
THE   3-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
      10,   2,   5,  12,  14,   6,  16,   8,  18,   3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =     0.41977D 01
     XB(  2) =    -0.36497D-01
     XB(  3) =     0.20988D 01
     XB(  4) =     0.40208D-15
     XB(  5) =    -0.69062D-15
     XB(  6) =     0.32089D 01
     XB(  7) =     0.64178D 01
     XB(  8) =     0.49697D 00
     XB(  9) =     0.99394D 00
     XB( 10) =     0.18921D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.58047D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =     0.13878D-16
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =     0.44409D-15
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.90756D 00
     THE   8-TH REDUCED COST =    -0.00000D 00
     THE   9-TH REDUCED COST =    -0.92437D-01
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.10000D 01
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.92437D-01
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.90756D 00
TENTATIVELY THE   2-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
     YR(  4) =    -0.10000D 01
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =     0.28011D 00
     YR(  8) =     0.00000D 00
     YR(  9) =    -0.28011D 00
     YR( 10) =     0.00000D 00
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
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        YR( 13) =      0.00000D 00
        YR( 14) =      0.00000D 00
        YR( 15) =      0.00000D 00
        YR( 16) =      0.00000D 00
        YR( 17) =     -0.28011D 00
        YR( 18) =      0.00000D 00
        YR( 19) =      0.28011D 00
    THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
        THE RATIO OF THE   4-TH NET PRICE TO YR(  4) =      0.44409D-15
        THE RATIO OF THE   9-TH NET PRICE TO YR(  9) =     -0.33000D 00
        THE RATIO OF THE  17-TH NET PRICE TO YR( 17) =     -0.33000D 00
    THE   2-TH VARIABLE IS LEAVING THE BASIS.
    THE   4-TH VARIABLE IS ENTERING THE BASIS.
    THE BASIC VARIABLES ARE NOW
         10,   4,   5,  12,  14,   6,  16,   8,  18,   3,
    THE VALUES OF THE BASIC VARIABLES ARE NOW
        XB(  1) =      0.41977D 01
        XB(  2) =      0.36497D-01
        XB(  3) =      0.20988D 01
        XB(  4) =      0.26962D-15
        XB(  5) =     -0.70683D-15
        XB(  6) =      0.32089D 01
        XB(  7) =      0.64178D 01
        XB(  8) =      0.49697D 00
        XB(  9) =      0.99394D 00
        XB( 10) =      0.18921D 00
    THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.58047D 01
    THE REDUCED COSTS
        THE   1-TH REDUCED COST =     0.27756D-16
        THE   2-TH REDUCED COST =    -0.22204D-15
        THE   3-TH REDUCED COST =    -0.00000D 00
        THE   4-TH REDUCED COST =    -0.00000D 00
        THE   5-TH REDUCED COST =    -0.00000D 00
        THE   6-TH REDUCED COST =    -0.00000D 00
        THE   7-TH REDUCED COST =    -0.90756D 00
        THE   8-TH REDUCED COST =    -0.00000D 00
        THE   9-TH REDUCED COST =    -0.92437D-01
        THE  10-TH REDUCED COST =    -0.00000D 00
        THE  11-TH REDUCED COST =    -0.10000D 01
        THE  12-TH REDUCED COST =    -0.00000D 00
        THE  13-TH REDUCED COST =    -0.10000D 01
        THE  14-TH REDUCED COST =    -0.00000D 00
        THE  15-TH REDUCED COST =    -0.10000D 01
        THE  16-TH REDUCED COST =    -0.00000D 00
        THE  17-TH REDUCED COST =    -0.92437D-01
        THE  18-TH REDUCED COST =    -0.00000D 00
        THE  19-TH REDUCED COST =    -0.90756D 00
THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
THE NONZERO VARIABLES ARE AS FOLLOWS:
        X( 10) =      0.41977D 01
        X(  4) =      0.36497D-01
        X(  5) =      0.20988D 01
        X( 12) =      0.26962D-15
        X( 14) =     -0.70683D-15
        X(  6) =      0.32089D 01
        X( 16) =      0.64178D 01
        X(  8) =      0.49697D 00
        X( 18) =      0.99394D 00
        X(  3) =      0.18921D 00
THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.58047D 01

SUPPLEMENTAL INFORMATION FROM THE MINI-MONTE CARLO STUDY

    THE GENERATED N(0,1) EPSILONS     SAMPLE NUMBER =      1
            -0.37319
            -0.57141
             0.04588
             0.10940
             0.29791
    THE PROBLEM P2 HAS BEEN SELECTED
    THE INVERSE OF XTX
        0.810298      -0.168497
       -0.168497       0.046521
    VALUE OF BETA0 TO COMPUTE RHS
       -0.594250       0.136933
    THE RHS, YMXB, FOR P1 OR P2
         0.067696
        -0.244174
         0.226595
        -0.039891
        -0.010226
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THE INITIAL VALUES OF THE BASIC VARIABLES
     XB(  1) =     -0.56988D 00
     XB(  2) =     -0.66336D-01
     XB(  3) =     -0.28494D 00
     XB(  4) =      0.80279D-16
     XB(  5) =      0.48323D-16
     XB(  6) =      0.54175D 00
     XB(  7) =      0.10835D 01
     XB(  8) =      0.10662D 00
     XB(  9) =      0.21323D 00
     XB( 10) =      0.42693D 00
THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.36343D 00
THE REDUCED COSTS
     THE   1-TH REDUCED COST =     0.27756D-16
     THE   2-TH REDUCED COST =    -0.22204D-15
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =    -0.00000D 00
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.90756D 00
     THE   8-TH REDUCED COST =    -0.00000D 00
     THE   9-TH REDUCED COST =    -0.92437D-01
     THE  10-TH REDUCED COST =    -0.00000D 00
     THE  11-TH REDUCED COST =    -0.10000D 01
     THE  12-TH REDUCED COST =    -0.00000D 00
     THE  13-TH REDUCED COST =    -0.10000D 01
     THE  14-TH REDUCED COST =    -0.00000D 00
     THE  15-TH REDUCED COST =    -0.10000D 01
     THE  16-TH REDUCED COST =    -0.00000D 00
     THE  17-TH REDUCED COST =    -0.92437D-01
     THE  18-TH REDUCED COST =    -0.00000D 00
     THE  19-TH REDUCED COST =    -0.90756D 00
TENTATIVELY THE   1-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =      0.00000D 00
     YR(  2) =      0.22204D-15
     YR(  3) =      0.00000D 00
     YR(  4) =      0.00000D 00
     YR(  5) =      0.00000D 00
     YR(  6) =      0.00000D 00
     YR(  7) =     -0.30644D 01
     YR(  8) =      0.00000D 00
     YR(  9) =      0.10644D 01
     YR( 10) =      0.00000D 00
     YR( 11) =      0.00000D 00
     YR( 12) =      0.00000D 00
     YR( 13) =      0.00000D 00
     YR( 14) =      0.00000D 00
     YR( 15) =     -0.10000D 01
     YR( 16) =      0.00000D 00
     YR( 17) =      0.30644D 01
     YR( 18) =      0.00000D 00
     YR( 19) =     -0.10644D 01
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE   7-TH NET PRICE TO YR(  7) =     -0.29616D 00
     THE RATIO OF THE  15-TH NET PRICE TO YR( 15) =     -0.10000D 01
     THE RATIO OF THE  19-TH NET PRICE TO YR( 19) =     -0.85263D 00
THE   1-TH VARIABLE IS LEAVING THE BASIS.
THE   7-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
       7,   4,   5,  12,  14,   6,  16,   8,  18,   3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.18596D 00
     XB(  2) =     -0.14245D-01
     XB(  3) =      0.27756D-16
     XB(  4) =      0.37193D 00
     XB(  5) =      0.86648D-17
     XB(  6) =      0.30005D 00
     XB(  7) =      0.60009D 00
     XB(  8) =      0.46190D-01
     XB(  9) =      0.92380D-01
     XB( 10) =      0.83651D-01
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.53220D 00
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.36082D-15
     THE   2-TH REDUCED COST =    -0.22204D-15
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =    -0.00000D 00
     THE   5-TH REDUCED COST =    -0.00000D 00
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        THE   6-TH REDUCED COST =    -0.00000D 00
        THE   7-TH REDUCED COST =    -0.00000D 00
        THE   8-TH REDUCED COST =    -0.00000D 00
        THE   9-TH REDUCED COST =    -0.40768D 00
        THE  10-TH REDUCED COST =    -0.29616D 00
        THE  11-TH REDUCED COST =    -0.10000D 01
        THE  12-TH REDUCED COST =    -0.00000D 00
        THE  13-TH REDUCED COST =    -0.10000D 01
        THE  14-TH REDUCED COST =    -0.00000D 00
        THE  15-TH REDUCED COST =    -0.70384D 00
        THE  16-TH REDUCED COST =    -0.00000D 00
        THE  17-TH REDUCED COST =    -0.10000D 01
        THE  18-TH REDUCED COST =    -0.00000D 00
        THE  19-TH REDUCED COST =    -0.59232D 00
    TENTATIVELY THE   2-TH BASIC VARIABLE IS LEAVING THE BASIS
    THE YRJ'S
    (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
    IT CAN BE ZERO OTHERWISE TOO.)
        YR(  1) =     -0.13878D-16
        YR(  2) =     -0.10000D 01
        YR(  3) =      0.00000D 00
        YR(  4) =      0.00000D 00
        YR(  5) =      0.00000D 00
        YR(  6) =      0.00000D 00
        YR(  7) =      0.00000D 00
        YR(  8) =      0.00000D 00
        YR(  9) =      0.18282D 00
        YR( 10) =     -0.91408D-01
        YR( 11) =      0.00000D 00
        YR( 12) =      0.00000D 00
        YR( 13) =      0.00000D 00
        YR( 14) =      0.00000D 00
        YR( 15) =      0.91408D-01
        YR( 16) =      0.00000D 00
        YR( 17) =      0.00000D 00
        YR( 18) =      0.00000D 00
        YR( 19) =     -0.18282D 00
    THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
        THE RATIO OF THE   2-TH NET PRICE TO YR(  2) =     -0.22204D-15
        THE RATIO OF THE  10-TH NET PRICE TO YR( 10) =     -0.32400D 01
        THE RATIO OF THE  19-TH NET PRICE TO YR( 19) =     -0.32400D 01
    THE   2-TH VARIABLE IS LEAVING THE BASIS.
    THE   2-TH VARIABLE IS ENTERING THE BASIS.
    THE BASIC VARIABLES ARE NOW
          7,   2,   5,  12,  14,   6,  16,   8,  18,   3,
    THE VALUES OF THE BASIC VARIABLES ARE NOW
        XB(  1) =      0.18596D 00
        XB(  2) =      0.14245D-01
        XB(  3) =      0.38305D-16
        XB(  4) =      0.37193D 00
        XB(  5) =      0.18154D-16
        XB(  6) =      0.30005D 00
        XB(  7) =      0.60009D 00
        XB(  8) =      0.46190D-01
        XB(  9) =      0.92380D-01
        XB( 10) =      0.83651D-01
    THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.53220D 00
    THE REDUCED COSTS
        THE   1-TH REDUCED COST =    -0.31919D-15
        THE   2-TH REDUCED COST =    -0.00000D 00
        THE   3-TH REDUCED COST =    -0.00000D 00
        THE   4-TH REDUCED COST =    -0.00000D 00
        THE   5-TH REDUCED COST =    -0.00000D 00
        THE   6-TH REDUCED COST =    -0.00000D 00
        THE   7-TH REDUCED COST =    -0.00000D 00
        THE   8-TH REDUCED COST =    -0.00000D 00
        THE   9-TH REDUCED COST =    -0.40768D 00
        THE  10-TH REDUCED COST =    -0.29616D 00
        THE  11-TH REDUCED COST =    -0.10000D 01
        THE  12-TH REDUCED COST =    -0.00000D 00
        THE  13-TH REDUCED COST =    -0.10000D 01
        THE  14-TH REDUCED COST =    -0.00000D 00
        THE  15-TH REDUCED COST =    -0.70384D 00
        THE  16-TH REDUCED COST =    -0.00000D 00
        THE  17-TH REDUCED COST =    -0.10000D 01
        THE  18-TH REDUCED COST =    -0.00000D 00
        THE  19-TH REDUCED COST =    -0.59232D 00
THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
THE NONZERO VARIABLES ARE AS FOLLOWS:
        X(  7) =     0.18596D 00
        X(  2) =     0.14245D-01
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        X(  5) =     0.38305D-16
        X( 12) =     0.37193D 00
        X( 14) =     0.18154D-16
        X(  6) =     0.30005D 00
        X( 16) =     0.60009D 00
        X(  8) =     0.46190D-01
        X( 18) =     0.92380D-01
        X(  3) =     0.83651D-01
THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.53220D 00

SUPPLEMENTAL INFORMATION FROM THE MINI-MONTE CARLO STUDY

    THE GENERATED N(0,1) EPSILONS     SAMPLE NUMBER =    2
             0.87619
             0.59391
            -0.35016
             0.51585
             1.51357
    THE PROBLEM P1 HAS BEEN SELECTED
    THE INVERSE OF XTX
        0.810298     -0.168497
       -0.168497      0.046521
    VALUE OF BETA0 TO COMPUTE RHS
        0.216974      0.113998
    THE RHS, YMXB, FOR P1 OR P2
       -0.531540
       -0.154642
        0.911404
        0.320132
       -0.545353
    THE INITIAL VALUES OF THE BASIC VARIABLES
        XB(  1) =     0.14477D 01
        XB(  2) =     0.25252D-02
        XB(  3) =    -0.19987D-15
        XB(  4) =     0.28955D 01
        XB(  5) =    -0.13871D-15
        XB(  6) =    -0.37899D 00
        XB(  7) =    -0.75799D 00
        XB(  8) =    -0.86256D 00
        XB(  9) =    -0.17251D 01
        XB( 10) =    -0.52871D 00
    THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.20619D 00
    THE REDUCED COSTS
        THE   1-TH REDUCED COST =    -0.31919D-15
        THE   2-TH REDUCED COST =    -0.00000D 00
        THE   3-TH REDUCED COST =    -0.00000D 00
        THE   4-TH REDUCED COST =    -0.00000D 00
        THE   5-TH REDUCED COST =    -0.00000D 00
        THE   6-TH REDUCED COST =    -0.00000D 00
        THE   7-TH REDUCED COST =    -0.00000D 00
        THE   8-TH REDUCED COST =    -0.00000D 00
        THE   9-TH REDUCED COST =    -0.40768D 00
        THE  10-TH REDUCED COST =    -0.29616D 00
        THE  11-TH REDUCED COST =    -0.10000D 01
        THE  12-TH REDUCED COST =    -0.00000D 00
        THE  13-TH REDUCED COST =    -0.10000D 01
        THE  14-TH REDUCED COST =    -0.00000D 00
        THE  15-TH REDUCED COST =    -0.70384D 00
        THE  16-TH REDUCED COST =    -0.00000D 00
        THE  17-TH REDUCED COST =    -0.10000D 01
        THE  18-TH REDUCED COST =    -0.00000D 00
        THE  19-TH REDUCED COST =    -0.59232D 00
    TENTATIVELY THE   9-TH BASIC VARIABLE IS LEAVING THE BASIS
    THE YRJ'S
    (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
    IT CAN BE ZERO OTHERWISE TOO.)
        YR(  1) =     0.44409D-15
        YR(  2) =     0.00000D 00
        YR(  3) =     0.00000D 00
        YR(  4) =     0.22204D-15
        YR(  5) =     0.00000D 00
        YR(  6) =     0.00000D 00
        YR(  7) =     0.00000D 00
        YR(  8) =     0.00000D 00
        YR(  9) =     0.15759D 01
        YR( 10) =     0.21207D 00
        YR( 11) =     0.00000D 00
        YR( 12) =     0.00000D 00
        YR( 13) =    -0.10000D 01
        YR( 14) =     0.00000D 00
        YR( 15) =    -0.21207D 00
```

(ETC.)

(RECONTINUED)

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     THE VALUES OF THE BASIC VARIABLES ARE NOW
          XB(  1) =     -0.41633D-16
          XB(  2) =      0.56469D-01
          XB(  3) =      0.73958D 00
          XB(  4) =      0.34616D 01
          XB(  5) =      0.18851D 00
          XB(  6) =      0.17308D 01
          XB(  7) =      0.26920D-16
          XB(  8) =      0.94254D-01
          XB(  9) =      0.14792D 01
          XB( 10) =      0.38392D 00
     THE CURRENT VALUE OF THE OBJECTIVE FUNCT.ON =     -0.25646D 01
     THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.11241D-14
          THE   2-TH REDUCED COST =    -0.00000D 00
          THE   3-TH REDUCED COST =    -0.00000D 00
          THE   4-TH REDUCED COST =    -0.44409D-15
          THE   5-TH REDUCED COST =    -0.00000D 00
          THE   6-TH REDUCED COST =    -0.00000D 00
          THE   7-TH REDUCED COST =    -0.00000D 00
          THE   8-TH REDUCED COST =    -0.00000D 00
          THE   9-TH REDUCED COST =    -0.00000D 00
          THE  10-TH REDUCED COST =    -0.00000D 00
          THE  11-TH REDUCED COST =    -0.10000D 01
          THE  12-TH REDUCED COST =    -0.77871D 00
          THE  13-TH REDUCED COST =    -0.00000D 00
          THE  14-TH REDUCED COST =    -0.72129D 00
          THE  15-TH REDUCED COST =    -0.10000D 01
          THE  16-TH REDUCED COST =    -0.00000D 00
          THE  17-TH REDUCED COST =    -0.22129D 00
          THE  18-TH REDUCED COST =    -0.10000D 01
          THE  19-TH REDUCED COST =    -0.27871D 00
THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
THE NONZERO VARIABLES ARE AS FOLLOWS:
          X(  7) =    -0.41633D-16
          X(  2) =     0.56469D-01
          X(  5) =     0.73958D 00
          X( 16) =     0.34616D 01
          X( 13) =     0.18851D 00
          X(  6) =     0.17308D 01
          X(  9) =     0.26920D-16
          X(  8) =     0.94254D-01
          X( 10) =     0.14792D 01
          X(  3) =     0.38392D 00
THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.25646D 01
SUPPLEMENTAL INFORMATION FROM THE MINI-MONTE CARLO STUDY

     THE GENERATED N(0,1) EPSILONS     SAMPLE NUMBER =    20
               0.41901
              -0.48994
              -1.15302
              -0.01590
              -1.66441
     THE PROBLEM P2 HAS BEEN SELECTED
     THE INVERSE OF XTX
         0.810298      -0.168497
        -0.168497       0.046521
     VALUE OF BETA0 TO COMPUTE RHS
         0.178028      -0.209520
     THE RHS, YMXB, FOR P1 OR P2
          0.475645
         -0.259401
         -0.698296
          0.943760
         -0.461707
     THE INITIAL VALUES OF THE BASIC VARIABLES
          XB(  1) =    -0.17131D-15
          XB(  2) =    -0.66271D-01
          XB(  3) =     0.12999D 01
          XB(  4) =    -0.10196D 01
          XB(  5) =     0.29647D 01
          XB(  6) =    -0.50980D 00
          XB(  7) =    -0.65143D-16
          XB(  8) =     0.14823D 01
          XB(  9) =     0.25997D 01
          XB( 10) =    -0.89844D 00
     THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =     -0.22724D 01
     THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.11241D-14
          THE   2-TH REDUCED COST =    -0.00000D 00
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     THE   3-TH REDUCED COST =     -0.00000D 00
     THE   4-TH REDUCED COST =     -0.44409D-15
     THE   5-TH REDUCED COST =     -0.00000D 00
     THE   6-TH REDUCED COST =     -0.00000D 00
     THE   7-TH REDUCED COST =     -0.00000D 00
     THE   8-TH REDUCED COST =     -0.00000D 00
     THE   9-TH REDUCED COST =     -0.00000D 00
     THE  10-TH REDUCED COST =     -0.00000D 00
     THE  11-TH REDUCED COST =     -0.10000D 01
     THE  12-TH REDUCED COST =     -0.77871D 00
     THE  13-TH REDUCED COST =     -0.00000D 00
     THE  14-TH REDUCED COST =     -0.72129D 00
     THE  15-TH REDUCED COST =     -0.10000D 01
     THE  16-TH REDUCED COST =     -0.00000D 00
     THE  17-TH REDUCED COST =     -0.22129D 00
     THE  18-TH REDUCED COST =     -0.10000D 01
     THE  19-TH REDUCED COST =     -0.27871D 00
TENTATIVELY THE   4-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS,
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =      0.51348D-15
     YR(  2) =      0.00000D 00
     YR(  3) =      0.00000D 00
     YR(  4) =     -0.11102D-14
     YR(  5) =      0.00000D 00
     YR(  6) =      0.00000D 00
     YR(  7) =      0.00000D 00
     YR(  8) =      0.00000D 00
     YR(  9) =      0.00000D 00
     YR( 10) =      0.00000D 00
     YR( 11) =     -0.10000D 01
     YR( 12) =      0.12997D 01
     YR( 13) =      0.00000D 00
     YR( 14) =     -0.29972D 00
     YR( 15) =      0.00000D 00
     YR( 16) =      0.00000D 00
     YR( 17) =     -0.12997D 01
     YR( 18) =      0.00000D 00
     YR( 19) =      0.29972D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE  11-TH NET PRICE TO YR( 11) =     -0.10000D 01
     THE RATIO OF THE  14-TH NET PRICE TO YR( 14) =     -0.24065D 01
     THE RATIO OF THE  17-TH NET PRICE TO YR( 17) =     -0.17026D 00
THE   4-TH VARIABLE IS LEAVING THE BASIS.
THE  17-TH VARIABLE IS ENTERING THE BASIS.
THE BASIC VARIABLES ARE NOW
       7,    2,    5,   17,   13,    6,    9,    8,   10,    3,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =      0.39224D 00
     XB(  2) =      0.43600D-01
     XB(  3) =      0.69886D 00
     XB(  4) =      0.78448D 00
     XB(  5) =      0.27098D 01
     XB(  6) =     -0.11102D-15
     XB(  7) =      0.72940D-16
     XB(  8) =      0.13549D 01
     XB(  9) =      0.13977D 01
     XB( 10) =     -0.17438D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =     -0.24460D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =     -0.99920D-15
     THE   2-TH REDUCED COST =     -0.00000D 00
     THE   3-TH REDUCED COST =     -0.00000D 00
     THE   4-TH REDUCED COST =     -0.00000D 00
     THE   5-TH REDUCED COST =     -0.00000D 00
     THE   6-TH REDUCED COST =     -0.00000D 00
     THE   7-TH REDUCED COST =     -0.00000D 00
     THE   8-TH REDUCED COST =     -0.00000D 00
     THE   9-TH REDUCED COST =     -0.00000D 00
     THE  10-TH REDUCED COST =     -0.00000D 00
     THE  11-TH REDUCED COST =     -0.82974D 00
     THE  12-TH REDUCED COST =     -0.10000D 01
     THE  13-TH REDUCED COST =     -0.00000D 00
     THE  14-TH REDUCED COST =     -0.67026D 00
     THE  15-TH REDUCED COST =     -0.10000D 01
     THE  16-TH REDUCED COST =     -0.17026D 00
     THE  17-TH REDUCED COST =     -0.00000D 00
     THE  18-TH REDUCED COST =     -0.10000D 01
     THE  19-TH REDUCED COST =     -0.32974D 00
TENTATIVELY THE  10-TH BASIC VARIABLE IS LEAVING THE BASIS
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     THE YRJ'S
     (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
     IT CAN BE ZERO OTHERWISE TOO.)
          YR(  1) =    -0.10000D 01
          YR(  2) =     0.00000D 00
          YR(  3) =     0.00000D 00
          YR(  4) =     0.66613D-15
          YR(  5) =     0.00000D 00
          YR(  6) =     0.00000D 00
          YR(  7) =     0.00000D 00
          YR(  8) =     0.00000D 00
          YR(  9) =     0.00000D 00
          YR( 10) =     0.00000D 00
          YR( 11) =     0.71013D 00
          YR( 12) =     0.18041D-15
          YR( 13) =     0.00000D 00
          YR( 14) =    -0.21013D 00
          YR( 15) =     0.00000D 00
          YR( 16) =    -0.71013D 00
          YR( 17) =     0.00000D 00
          YR( 18) =     0.00000D 00
          YR( 19) =     0.21013D 00
     THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
          THE RATIO OF THE   1-TH NET PRICE TO YR(  1) =    -0.99920D-15
          THE RATIO OF THE  14-TH NET PRICE TO YR( 14) =    -0.31897D 01
          THE RATIO OF THE  16-TH NET PRICE TO YR( 16) =    -0.23976D 00
     THE  10-TH VARIABLE IS LEAVING THE BASIS.
     THE   1-TH VARIABLE IS ENTERING THE BASIS.
     THE BASIC VARIABLES ARE NOW
            7,   2,   5,  17,  13,   6,   9,   8,  10,   1,
     THE VALUES OF THE BASIC VARIABLES ARE NOW
          XB(  1) =     0.39224D 00
          XB(  2) =     0.43600D-01
          XB(  3) =     0.69886D 00
          XB(  4) =     0.78448D 00
          XB(  5) =     0.27098D 01
          XB(  6) =    -0.12796D-15
          XB(  7) =     0.85041D-16
          XB(  8) =     0.13549D 01
          XB(  9) =     0.13977D 01
          XB( 10) =     0.17438D 00
     THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.24460D 01
     THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.00000D 00
          THE   2-TH REDUCED COST =    -0.00000D 00
          THE   3-TH REDUCED COST =    -0.12490D-15
          THE   4-TH REDUCED COST =    -0.00000D 00
          THE   5-TH REDUCED COST =    -0.00000D 00
          THE   6-TH REDUCED COST =    -0.00000D 00
          THE   7-TH REDUCED COST =    -0.00000D 00
          THE   8-TH REDUCED COST =    -0.00000D 00
          THE   9-TH REDUCED COST =    -0.00000D 00
          THE  10-TH REDUCED COST =    -0.00000D 00
          THE  11-TH REDUCED COST =    -0.82974D 00
          THE  12-TH REDUCED COST =    -0.10000D 01
          THE  13-TH REDUCED COST =    -0.00000D 00
          THE  14-TH REDUCED COST =    -0.67026D 00
          THE  15-TH REDUCED COST =    -0.10000D 01
          THE  16-TH REDUCED COST =    -0.17026D 00
          THE  17-TH REDUCED COST =    -0.00000D 00
          THE  18-TH REDUCED COST =    -0.10000D 01
          THE  19-TH REDUCED COST =    -0.32974D 00
THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
THE NONZERO VARIABLES ARE AS FOLLOWS:
          X(  7) =     0.39224D 00
          X(  2) =     0.43600D-01
          X(  5) =     0.69886D 00
          X( 17) =     0.78448D 00
          X( 13) =     0.27098D 01
          X(  6) =    -0.12796D-15
          X(  9) =     0.85041D-16
          X(  8) =     0.13549D 01
          X( 10) =     0.13977D 01
          X(  1) =     0.17438D 00
THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.24460D 01


THE AUXILIARY LEAST SQUARES ESTIMATE, BETAO, OF THE REGRESSION PARAMETER VECTOR, BETA
       LEAST SQUARES ESTIMATE OF BETA(  1) =        3.227002
       LEAST SQUARES ESTIMATE OF BETA(  2) =       -1.159747
MRS. A'S ANSWER :     THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS:
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```
     L1 ESTIMATE OF BETA(  1) =      3.416213
     L1 ESTIMATE OF BETA(  2) =     -1.123249
THE RESIDUALS, R(I),I=1,NOBS
             2.098826
             3.208877
             0.000000
             0.496969
             0.000000
THE SUM OF THE ABSOLUTE RESIDUALS =      5.804672
THE MAXIMUM ABSOLUTE RESIDUAL =      3.208877
AUXILIARY RESULTS OF THE MINI-MONTE CARLO STUDY

  VALUES OF DELTA BETA STAR
    SAMPLE NUMBER =  1      -0.51060    0.12269
    SAMPLE NUMBER =  2       0.63765   -0.02243
    SAMPLE NUMBER =  3       0.94741   -0.45843
    SAMPLE NUMBER =  4       0.05573   -0.14477
    SAMPLE NUMBER =  5      -0.08930   -0.16773
    SAMPLE NUMBER =  6      -1.10362    0.22395
    SAMPLE NUMBER =  7      -0.86781   -0.04936
    SAMPLE NUMBER =  8      -1.24035    0.54421
    SAMPLE NUMBER =  9       0.44742    0.24662
    SAMPLE NUMBER = 10      -1.03640    0.36600
    SAMPLE NUMBER = 11       0.83504    0.01230
    SAMPLE NUMBER = 12       0.32935   -0.04047
    SAMPLE NUMBER = 13      -1.27719    0.27656
    SAMPLE NUMBER = 14      -0.58869    0.33108
    SAMPLE NUMBER = 15      -0.04885    0.14684
    SAMPLE NUMBER = 16       0.98576   -0.03155
    SAMPLE NUMBER = 17      -1.01343    0.04536
    SAMPLE NUMBER = 18       1.70310   -0.28731
    SAMPLE NUMBER = 19      -0.37732   -0.04003
    SAMPLE NUMBER = 20       0.00365   -0.25312
  ESTIMATED COVARIANCE OF DELTA BETA STAR
       0.774642    -0.148480
      -0.148480     0.063634
  SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER ALL SAMPLES =      48.526882

AUXILIARY RESULT:  SIGMA HAT 5 =      2.425026
MAIN RESULTS OF THE MINI-MONTE CARLO STUDY
ESTIMATED VALUE OF SIGMA (SIGMA HAT 4) =      2.392353
ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR (BETA) USING THIS ESTIMATE OF SIGM:
      4.433552    -0.849803
?
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APPENDIX C. PROGRAM LISTING

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C
C      MRS.A :
C
C      MINIMIZES SUM OF ABSOLUTE RESIDUALS.
C
C      THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
C      THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
C
C
C    L1 ESTIMATION -
C
C    THIS PROGRAM USES THE DUAL SIMPLEX ALGORITHM TO IMPLEMENT
C    THE L1 ESTIMATION PROCEDURE OUTLINED IN THE PAPER:
C    HARTLEY AND SIELKEN 'TWO LINEAR PROGRAMMING ALGORITHMS
C    FOR UNBIASED ESTIMATION OF LINEAR MODELS',
C    1973, JASA, VOL 68, NO.343, PAGES 639-641.
C
C
C    THE VARIANCE OF THIS UNBIASED L1 ESTIMATOR IS ESTIMATED
C    USING A  MINI MONTE CARLO APPROACH.
C    THE SUM OF THE ABSOLUTE RESIDUALS, R(I), IS MINIMIZED SUBJECT TO
C      -R .LE. Y-XB .LE. R
C      B UNRESTRICTED,
C      R .GE. 0.
C
C    WHERE:
C           Y = A VECTOR OF NOBS OBSERVATIONS
C           X = A NOBS BY IP MATRIX OF CONSTANTS
C           B = AN IP BY 1 VECTOR OF UNKNOWN PARAMETERS.
C
C
C
C    THE DUAL SIMPLEX ALGORITHM . . . .
C
C     THE LINEAR PROGRAMMING PROBLEM IS PUT INTO THE FORM
C           MAX CX
C        SUBJECT TO
C           AX = BRHS
C           X GREATER THAN OR EQUAL TO 0
C     WHERE
C           BRHS IS A COLUMN OF CONSTANTS
C           A IS AN M-BY-N MATRIX OF CONSTANTS
C
C     THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
C     A1,BRHS1,B1,XB1,AND Y1 RESPECTIVELY.
C
C
C     THE FOLLOWING PROCEDURE WAS DEVELOPED BY :
C          D.N. BOOK
C          J.B. BOOKER
C          H.O. HARTLEY
C          R.L. SIELKEN, JR.
C     INSTITUTE OF STATISTICS
C     TEXAS A&M UNIVERSITY
C     COLLEGE STATION, TEXAS 77843
C
C     INQUIRIES AND COMMENTS SHOULD BY ADDRESSED TO :
C          ROBERT L. SIELKEN, JR.
C
C     THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY
C     ACKNOWLEDGED.
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION C(80),BRHS1(41),A1(41,80),INBASE(40)
      DIMENSION DELTA(10),WT(20),TITLE(20)
      DIMENSION XB1(41),Y1(41),REDCOS(80),ISTAT(80),YR(80)
      DIMENSION XBINT(80),ESTAR(20),DELB(10,100),CAPB1(10),CAPB2(10)
      DIMENSION DBX(80),VARB(10,10),VAROBS(10,10),SUM(10)
      DIMENSION X(20,10),Y(20),DB1(10),DB2(10),DBETA0(10)
      DIMENSION BHAT(10),BETA0(10)
      COMMON/XTXIM/XTX(41,41)
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      COMMON/NFEED/NSEED
      COMMON/ICONST/ITERA,ITIP,MP1,IP1P2
      DOUBLE PRECISION B1INV(41,41)
C
C      THE DIMENSIONED ARRAYS HAVE THE FOLLOWING DIMENSIONS:
C           C(N),BRHS1(M+1),A1(M+1,N),INBASE(M),ISTAT(N)
C           B1INV(M+1,M+1),XB1(M+1),Y1(M+1),REDCOS(N),YR(N)
C           XBINT(N),ESTAR(NOBS),DELB(IP,ISAM),CAPB1(IP),CAPB2(IP)
C           SUM(IP),WT(NOBS)
C           VAROBS(IP,IP),VARB(IP,IP),DBX(N)
C           X(NOBS,IP),Y(NOBS),DB1(IP),DB2(IP),BETA0(IP),BHAT(IP)
C           DBETA (IP)
C
C           WHERE:
C                  IP = THE NUMBER OF PARAMETERS
C                  NOBS = THE NUMBER OF OBSERVATIONS
C                  ISAM = THE NUMBER OF MONTE CARLO SAMPLES
C                  M = THE NUMBER OF CONSTRAINTS = 2*NOBS
C                  N = THE NUMBER OF VARIABELES = 2*IP + 3*NOBS
C
C
C      THE FOLLOWING 'TOLERANCES' ARE USED IN THE ALGORITHM.
C      THEY WOULD BE ZERO EXCEPT FOR THE NUMERICAL INACCURACY OF THE
C      COMPUTER
C           TOLR1 : IF THE MAX REDUCED COST IS LESS THAN
C                   TOLR1 THEN ALL REDUCED COSTS ARE CONSIDERED TO BE
C                   NON-POSITIVE.
C           TOLR2 : ANY COMPONENT Y(I,J) >OR = TOLR2 IS
C                   CONSIDERED NON-NEGATIVE
C           TOLR3 : IF A BASIC VARIABLE IS > OR = TOLR3, IT IS
C                   CONSIDERED TO BE NON-NEGATIVE
C
      TOLR1=1.0D-07
      TOLR2=-1.0D-07
      TOLR3=-1.0D-07
C
C      THE INPUT - CARD NUMBER ONE.
C
C      NOBS = NUMBER OF OBSERVATIONS,
C      IP = NUMBER OF PARAMETERS,
C      ISAM = NUMBER OF SAMPLES,
C      NSEED = 10 DIGIT RANDOM NUMBER LESS THAN 2147483647,
C      AND IWRIT1, IWRIT2, IWRIT3, IWRIT4, IWRIT5, IWRIT6; AND IOPTN
C      WHERE:
C          THE IWRIT VARIABLES INDICATE WHETHER OR NOT A
C          WRITE STATEMENT IS TO BE PRINTED.
C      IF IWRIT1=1, THEN PRINTING OF THE MAIN DUAL SIMPLEX OCCURS.
```

```
C
C      MRS.A :
C
C      MINIMIZES SUM OF ABSOLUTE RESIDUALS.
C
C      THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
C      THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
C
C
C   L1 ESTIMATION -
C
C   THIS PROGRAM USES THE DUAL SIMPLEX ALGORITHM TO IMPLEMENT
C   THE L1 ESTIMATION PROCEDURE OUTLINED IN THE PAPER:
C   HARTLEY AND SIELKEN 'TWO LINEAR PROGRAMMING ALGORITHMS
C   FOR UNBIASED ESTIMATION OF LINEAR MODELS',
C   1973, JASA, VOL 68, NO.343, PAGES 639-641.
C
C
C   THE VARIANCE OF THIS UNBIASED L1 ESTIMATOR IS ESTIMATED
C   USING A  MINI MONTE CARLO APPROACH.
C   THE SUM OF THE ABSOLUTE RESIDUALS, R(I), IS MINIMIZED SUBJECT TO
C     -R .LE. Y-XB .LE. R
C     B UNRESTRICTED,
C     R .GE. 0.
C
C   WHERE:
C          Y = A VECTOR OF NOBS OBSERVATIONS
C          X = A NOBS BY IP MATRIX OF CONSTANTS
C          B = AN IP BY 1 VECTOR OF UNKNOWN PARAMETERS.
C
C
C
C   THE DUAL SIMPLEX ALGORITHM . . . .
C
C    THE LINEAR PROGRAMMING PROBLEM IS PUT INTO THE FORM
C         MAX CX
C      SUBJECT TO
C         AX = BRHS
C         X GREATER THAN OR EQUAL TO 0
C    WHERE
C         BRHS IS A COLUMN OF CONSTANTS
C         A IS AN M-BY-N MATRIX OF CONSTANTS
C
C    THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
C    A1,BRHS1,B1,XB1,AND Y1 RESPECTIVELY.
C
C
C    THE FOLLOWING PROCEDURE WAS DEVELOPED BY :
C        D.N. BOOK
C        J.B. BOOKER
C        H.O. HARTLEY
C        R.L. SIELKEN, JR.
C    INSTITUTE OF STATISTICS
C    TEXAS A&M UNIVERSITY
C    COLLEGE STATION, TEXAS 77843
C
C    INQUIRIES AND COMMENTS SHOULD BY ADDRESSED TO :
C        ROBERT L. SIELKEN, JR.
C
C    THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY
C    ACKNOWLEDGED.
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION C(80),BRHS1(41),A1(41,80),INBASE(40)
      DIMENSION DELTA(10),WT(20),TITLE(20)
      DIMENSION XB1(41),Y1(41),REDCOS(80),ISTAT(80),YR(80)
      DIMENSION XBINT(80),ESTAR(20),DELB(10,100),CAPB1(10),CAPB2(10)
      DIMENSION DBX(80),VARB(10,10),VARDBS(10,10),SUM(10)
      DIMENSION X(20,10),Y(20),DB1(10),DB2(10),DBETA0(10)
      DIMENSION BHAT(10),BETA0(10)
      COMMON/XTXIM/XTX(41,41)
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      COMMON/NFEED/NSEED
      COMMON/ICONST/ITERA,ITIP,MP1,IP1P2
      DOUBLE PRECISION B1INV(41,41)
C
C     THE DIMENSIONED ARRAYS HAVE THE FOLLOWING DIMENSIONS:
C          C(N),BRHS1(M+1),A1(M+1,N),INBASE(M),ISTAT(N)
C          B1INV(M+1,M+1),XB1(M+1),Y1(M+1),REDCOS(N),YR(N)
C          XBINT(N),ESTAR(NOBS),DELB(IP,ISAM),CAPB1(IP),CAPB2(IP)
C          SUM(IP),WT(NOBS)
C          VARDBS(IP,IP),VARB(IP,IP),DBX(N)
C          X(NOBS,IP),Y(NOBS),DB1(IP),DB2(IP),BETA0(IP),BHAT(IP)
C          DBETA (IP)
C
C          WHERE:
C                IP = THE NUMBER OF PARAMETERS
C                NOBS = THE NUMBER OF OBSERVATIONS
C                ISAM = THE NUMBER OF MONTE CARLO SAMPLES
C                M = THE NUMBER OF CONSTRAINTS = 2*NOBS
C                N = THE NUMBER OF VARIABELES = 2*IP + 3*NOBS
C
C
C     THE FOLLOWING 'TOLERANCES' ARE USED IN THE ALGORITHM.
C     THEY WOULD BE ZERO EXCEPT FOR THE NUMERICAL INACCURACY OF THE
C     COMPUTER
C          TOLR1 : IF THE MAX REDUCED COST IS LESS THAN
C                  TOLR1 THEN ALL REDUCED COSTS ARE CONSIDERED TO BE
C                  NON-POSITIVE.
C          TOLR2 : ANY COMPONENT Y(I,J) >OR = TOLR2 IS
C                  CONSIDERED NON-NEGATIVE
C          TOLR3 : IF A BASIC VARIABLE IS > OR = TOLR3, IT IS
C                  CONSIDERED TO BE NON-NEGATIVE
C
      TOLR1=1.0D-07
      TOLR2=-1.0D-07
      TOLR3=-1.0D-07
C
C     THE INPUT - CARD NUMBER ONE.
C
C     NOBS = NUMBER OF OBSERVATIONS,
C     IP = NUMBER OF PARAMETERS,
C     ISAM = NUMBER OF SAMPLES,
C     NSEED = 10 DIGIT RANDOM NUMBER LESS THAN 2147483647,
C     AND IWRIT1, IWRIT2, IWRIT3, IWRIT4, IWRIT5, IWRIT6; AND IOPTN
C     WHERE:
C          THE IWRIT VARIABLES INDICATE WHETHER OR NOT A
C          WRITE STATEMENT IS TO BE PRINTED.
C     IF IWRIT1=1, THEN PRINTING OF THE MAIN DUAL SIMPLEX OCCURS.
```

```
C        IF IWRIT2=1, THEN PRINTING OF INTERMEDIATE STEPS OCCURS.
C        IF IWRIT3=1, THEN PRINTING OF THE MAIN DUAL SIMPLEX STEPS
C                     IN THE MONTE CARLO SAMPLES OCCURS.
C        IF IWRIT4=1, THEN PRINTING OF THE INTERMEDIATE MONTE CARLO
C                     MONTE CARLO STEPS OCCURS.
C        IF IWRIT5=1, THEN PRINTING OF THE INPUTTED VALUES OF
C                     X AND Y OCCURS.
C        IF IWRIT6=1, THEN PRINTING OF THE INTERMEDIATE CALCULATIONS
C                     FOR THE ESTIMATED VARIANCE OF B OCCURS.
C        IF ANY IWRIT VARIABLE = 0, THEN NO PRINTING OCCURS.
C
C        IOPTN = 1, IF THE GENERATED EPSILONS IN THE MINI-MONTE CARLO
C                   STUDY ARE DISTRIBUTED AS NORMAL RANDOM VARIABLES.
C        IOPTN = 2, IF THE GENERATED EPSILONS IN THE MINI-MONTE CARLO
C                   STUDY ARE DISTRIBUTED AS DOUBLE EXPONENTIALS.
C        IOPTN = 3, IF THE GENERATED EPSILONS IN THE MINI-MONTE CARLO
C                   STUDY ARE DISTRIBUTED AS UNIFORM RANDOM VARIABLES.
C
C        IWT = 1, IF THE RESIDUALS ARE ASSIGNED WEIGHT COEFFICIENTS.
C            = 0, IF NO WEIGHTS ARE ASSIGNED TO RESIDUALS.
C
      READ(5,100) NOBS,IP,ISAM,NSEED,IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,
     *IWRIT6,IOPTN,IWT
100   FORMAT(3I5,I11,6I2,I2,I2)
      ITERA=0
      SUMOBJ=0.D0
C
C          PRINTING OF HEADINGS AND INFORMATION
C
      WRITE(6,2321)
 2321 FORMAT(1H1,19X,'MRS. A !',/,/,6X,
     *'MINIMIZES SUM OF ABSOLUTE RESIDUALS.',/,/,6X,
     *'      THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING',
     */,6X,'THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.',/,
     *6X,'IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN',
     */,6X,'ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED',/,6X,
     *'REGRESSION PARAMETERS.',/,6X,'      UNBIASED ESTIMATES OF THE ',
     *'REGRESSION ',/,6X,'PARAMETERS ARE OBTAINED USING THE PROCEDURE'
     *,/,6X,'DESCRIBED IN A PAPER BY H.O. HARTLEY AND R.L. SIELKEN,JR,',
     */,6X,'"TWO LINEAR PROGRAMMING ALGORITHMS FOR UNBIASED ESTIMATION',
     */,6X,'OF LINEAR MODELS", 1973, JASA, VOL. 68, PAGES 639-41.',/,
     */,6X,'      THE FOLLOWING PROCEDURE DEVELOPED BY :',/,15X,
     *'D.N. BOOK',/,15X,'J.B. BOOKER',/,15X,'H.O. HARTLEY',/,15X,
     *'R.L. SIELKEN, JR.',/,11X,'INSTITUTE OF STATISTICS',/,11X,
     *'TEXAS A & M UNIVERSTIY',/,11X,'COLLEGE STATION, TEXAS 77843',
     */,/,11X,'INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:',/,
     *15X,'ROBERT L. SIELKEN, JR.',/,/,/,6X,'      THE SUPPORT OF THE ',
     *'OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED.')
C
C          INPUT - SECOND CARD GROUP, READ ONLY IF RESIDUALS ARE
C                  ASSIGNED WEIGHTS (IWT=1).
C
C          USER SUPPLIED WEIGHT COEFFICIENTS TO ASSIGN WEIGHTS TO THE
C          RESIDUALS (OBJECTIVE FUNCTION WEIGHTS) INPUTED HERE, 16 PER
C          CARD IN F5.1 FORMAT.
C
      IF(IWT.EQ.1) READ(5,2329) (WT(J),J=1,NOBS)
 2329 FORMAT(8F10.5)
C
C          INDEXING QUANTITIES DEFINED BELOW. . .
C
      M=2*NOBS
      N=2*IP + 3*NOBS
C
C         M = THE NUMBER OF CONSTRAINTS  (NOT INCLUDING THE OBJECTIVE
C             FUNCTION)
C         N = THE NUMBER OF VARIABLES
C
      MP1=M+1
      NRV=2*IP + NOBS
      NOBS3=3*NOBS
      ITIP=2*IP
C
C         C IS A ROW OF CONSTANTS. THE J-TH ELEMENT OF C IS THE
C         COEFFICIENT OF THE J-TH VARIABLE IN THE OBJECTIVE FUNCTION
C         C(J) IS DEFINED BELOW.
C
C
      IF (IWT.EQ.0) GO TO 2111
      GO TO 2112
 2111 DO 2110 J=1,NOBS
 2110 WT(J)=1.D0
 2112 DO 2010 J=1,N
      C(J)=0.D0
      IF (J.GT.ITIP.AND.J.LE.NRV) C(J)=-WT(J-ITIP)
      IF (J.GT.NRV) C(J)=0.D0
 2010 CONTINUE
C
C          THE INPUT - CARD GROUP THREE.
C
C          READ IN VALUES OF Y, A VECTOR OF NOBS VALUES.
C
      READ(5,2011) (Y(J),J=1,NOBS)
 2011 FORMAT (8F10.5)
C
C          THE INPUT - CARD GROUP FOUR.
C
C          READ IN THE VALUES OF X, AN NOBS BY IP MATRIX.
C
      DO 2002 I=1,NOBS
 2002 READ(5,2012) (X(I,J), J=1,IP)
 2012 FORMAT(8F10.5)
C
C          READ IN AND PRINT OUT USER SUPPLIED TITLE CARD BELOW.
C
      READ(5,2328) (TITLE(I),I=1,20)
 2328 FORMAT(20A4)
      WRITE(6,2322) (TITLE(I),I=1,20)
 2322 FORMAT(1H1,/,6X ,20A4,/)
C
C          END OF INPUT.
C
C
C
C          PRINTING OF THE INPUTTED QUANTITIES.
C
      WRITE(6,2300) NOBS,IP,ISAM,NSEED
 2300 FORMAT(1H0,5X,'NUMBER OF OBSERVATIONS = ',I5,/,
     *6X,'NUMBER OF PARAMETERS = ',I5,/,
     *6X,'THE SAMPLE SIZE FOR THE MINI-MONTE CARLO STUDY = ',I5,/,
```

```
     *6X,'USER SUPPLIED INITIAL RANDOM INTEGER: NSEED = ',I11)
C
C         OPTIONAL PRINTOUT OF X AND Y WHEN IWRIT5=1.
C
      IF(IWRIT5.EQ.0) GO TO 2305
      WRITE(6,2324)
 2324 FORMAT(1H1,5X,'THE LINEAR REGRESSION IS Y = X*BETA + EPSILON',/,
     *6X,'WHERE',/,6X,'Y CONTAINS THE OBSERVATIONS,'/,6X,
     *'BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS,'
     *,/,6X,'THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA',
     *'/,6X,'CORRESPONDING TO THE I-TH OBSERVATION, AND',/,/,6X,
     *'EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE '
     *,/,6X,'SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM',
     */,6X,'X*BETA.')
      WRITE(6,2306)
 2306 FORMAT(1H0,10X,'THE Y VECTOR')
      DO 2307 I=1,NOBS
 2307 WRITE(6,2308) Y(I)
 2308 FORMAT(1H ,15X,F16.5)
      WRITE(6,2301)
 2301 FORMAT(1H0,10X,'THE X MATRIX')
      DO 2302 I=1,NOBS
 2302 WRITE(6,2303) (X(I,J), J=1,IP)
 2303 FORMAT(1H ,15X,16F7.1)
      IF(IWT.EQ.0) GO TO 2316
      WRITE(6,2313)
 2313 FORMAT(1H0,10X,'THE RESIDUAL WEIGHT COEFFICIENTS')
      DO 2315 I=1,NOBS
 2315 WRITE(6,2314) WT(I)
 2314 FORMAT(1H ,15X,F16.5)
 2316 CONTINUE
 2305 CONTINUE
C
C         CONSTRUCT THE CONSTRAINT MATRIX, THE A1 MATRIX:
C                            0    0   -1    0    0
C                           -X    X   -I    I    0
C                            X   -X   -I    0    I
C
      DO 21 K=1,N
21    A1(1,K)=-C(K)
      DO 2001 I=1,NOBS
      II=I+1
      III=II+NOBS
      DO 2003 J=1,IP
      A1(II,J)=-X(I,J)
      A1(III,J)=X(I,J)
      A1(II,J+IP)=X(I,J)
      A1(III,J+IP)=-X(I,J)
 2003 CONTINUE
C
C         CONSTRUCT THE IDENTITY MATRICES FOR U AND SLACK VARS.
C
      DO 2004 J=1,NOBS
      A1(II,NRV+J)=0.D0
      A1(III,NRV+J)=0.D0
      A1(II,NRV+NOBS+J)=0.D0
      A1(III,NRV+NOBS+J)=0.D0
      A1(II,ITIP+J)=0.D0
 2004 A1(III,ITIP+J)=0.D0
      A1(II,ITIP+I)=-1.D0
      A1(III,ITIP+I)=-1.D0
      A1(II,NRV+I)=1.D0
      A1(III,ITIP+M+I)=1.D0
 2001 CONTINUE
C
C
C         INBASE IS A SET OF M INTEGER VARIABLES WHICH INDICATE THE
C         COMPOSITION OF THE CURRENT BASIS B.  FOR EXAMPLE,
C              INBASE(K) = 7  IMPLIES THAT THE K-TH COLUMN IN THE BASIS
C                             CORRESPONDS TO THE 7-TH VARIABLE
C
      DO 2005 K=1,M
 2005 INBASE(K)=NRV + K
C
C         ISTAT INDICATES THE BASIC STATUS OF EACH VARIABLE
C              ISTAT(K) = 1  IMPLIES THAT THE K-TH VARIABLE IS IN THE
C                            BASIS
C              ISTAT(K) = 0  IMPLIES THAT THE K-TH VARIABLE IS NOT IN THE
C                            BASIS
C
      DO 2006 J=1,NRV
 2006 ISTAT(J)=0
      DO 2007 J=1,M
 2007 ISTAT(NRV+J)=1
C
C        WRITE OUT THE INITIAL PROBLEM AND THE INITIAL BASIC VARIABLES
C
      IF(IWRIT1.EQ.0) GO TO 2320
      WRITE(6,2325)
 2325 FORMAT(1H1,5X,'SUPPLEMENTAL INFORMATION FROM THE LINEAR PROGRAMM'
     *,'ING PROBLEM DETERMINATION OF THE ESTIMATE OF',/,1H,,'THE ',
     *'REGRESSION PARAMETER VECTOR, BETA.')
      WRITE(6,500)
500   FORMAT(1H0,'     THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED'
     *)
      WRITE(6,501)
501   FORMAT(1H0,'          THE OBJECTIVE FUNCTION COEFFICIENTS')
      DO 502 LM=1,N
502   WRITE(6,503) LM,C(LM)
503   FORMAT(1H ,15X,'C(',I3,') = ',F12.5)
      WRITE(6,508)
508   FORMAT(1H0,'          THE CONSTRAINT MATRIX A')
      DO 509 LM=1,M
      LMM=LM+1
509   WRITE(6,510) (A1(LMM,I),I=1,N)
510   FORMAT(1H ,5X,16(F7.1))
      WRITE(6,511)
511   FORMAT(1H0,'          THE VARIABLES INITIALLY IN THE BASIS')
      WRITE(6,512)(INBASE(I),I=1,M)
512   FORMAT(1H ,15X,25(I2,','))
 2320 CONTINUE
C
C         THE M+1 BY M+1 MATRIX B1INV IS THE INVERSE OF THE REVISED
C         SIMPLEX BASIS MATRIX B1.  SEE NOTES
C         THE FOLLOWING STATEMENTS CONSTRUCT THE INITIAL MATRIX B1INV
C
      DO 10 II=1,MP1
      DO 12 L=2,MP1
12    B1INV(L,II)=0.D0
10    B1INV(II,II)= 1.D0
```

```
      B1INV(1,1)=1.D0
      DO 11 I=1,M
      III=I+1
      KKK=INBASE(I)
11    B1INV(1,III)=-C(KKK)
C
C        CALL SUBROUTINE CONST TO CALCULATE XB1.
C
      CALL CONST(Y,X,IP,NOBS,XB1,BETAO,B1INV,A1)
300   CONTINUE
      IF(IWRIT1.EQ.0) GO TO 3555
      WRITE(6,513)
513   FORMAT(1H0,'          THE INITIAL VALUES OF THE BASIC VARIABLES')
      DO 515 I=2,MP1
      II=I-1
515   WRITE(6,514) II,XB1(I)
514   FORMAT(1H ,15X,'XB(',I3,') = ',E15.5)
 3555 CONTINUE
      IF(IWRIT1.EQ.1) WRITE(6,855) XB1(1)
855   FORMAT(1H0,10X,'THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = ',E1
     *5.5)
350   CONTINUE
C
C        START THE DUAL SIMPLEX ALGORITHM
C
C
C        COMPUTE THE REDUCED COSTS:  C(J) -Z(J)
C        THE J-TH REDUCED COST IS DENOTED BY REDCOS(J)
C
      DO 23 J=1,N
      R=0.D0
      IF(ISTAT(J) .EQ. 1) GO TO 1021
      DO 22 K=1,MP1
22    R=R+B1INV(1,K)*A1(K,J)
1021  REDCOS(J)=-R
23    CONTINUE
      IF(IWRIT2.EQ.0) GO TO 2330
      WRITE(6,1030)
1030  FORMAT(1H0,'          THE REDUCED COSTS')
      DO 1031 J=1,N
1031  WRITE(6,1032) J,REDCOS(J)
1032  FORMAT(1H ,15X,'THE ',I3,'-TH REDUCED COST = ',E15.5)
 2330 CONTINUE
C
C        FIND THE MAXIMUM REDUCED COST
C             RMAX = THE MAXIMUM REDUCED COST
C
      RMAX=REDCOS(1)
      DO 24 J=2,N
      IF(REDCOS(J) .LE. RMAX) GO TO 24
      RMAX=REDCOS(J)
24    CONTINUE
C
C        IF RMAX IS >OR= TOLR1  THEN THE OPTIMALITY CHARACTERISTIC IS
C        NOT PRESENT
C
      IF(RMAX .GE. TOLR1)   GO TO 402
C
C        IS THE CURRENT SOLUTION FEASIBLE?
C        DETERMINE THE SMALLEST BASIC VARIABLE VALUE
C
C           IRMIN= THE INDEX OF THE BASIC VARIABLE WITH THE SMALLEST
C                VALUE.
C           (THE INDEX IRMIN INCLUDES THE OBJECTIVE FUNCTION AS
C            NUMBER ONE.)
C
      IRMIN=0
      RMIN=0.D0
      DO 1001 I=2,MP1
      W=XB1(I)-RMIN
      IF(W .GE. TOLR3)    GO TO 1001
      RMIN=XB1(I)
      IRMIN=I
1001  CONTINUE
C
C        IF RMIN EXCEEDS TOLR3   THEN THE CURRENT SOLUTION IS FEASIBLE
C        AND CONSEQUENTLY OPTIMAL
C
      IF(RMIN .GT.TOLR3)    GO TO 401
C
C        THE CURRENT SOLUTION IS STILL INFEASIBLE
C        THE IRMIN-TH VARIABLE IS TO LEAVE THE BASIS
C
      IL=IRMIN-1
      IF (IWRIT2.EQ.1) WRITE(6,1040) IL
1040  FORMAT(1H0, 10X,'TENTATIVELY THE ',I3,'-TH BASIC VARIABLE IS LEAVI
     *NG THE BASIS')
C
C        IS THE PROBLEM ITSELF INFEASIBLE?
C
      DO 26 J=1,N
      YR(J)=0.D0
      IF(ISTAT(J) .EQ. 1) GO TO 26
      DO 1002 LL=2,MP1
1002  YR(J)=YR(J)+B1INV(IRMIN,LL)*A1(LL,J)
26    CONTINUE
      IF(IWRIT2.EQ.0) GO TO 2340
      WRITE(6,1036)
1036  FORMAT(1H0,10X,'THE YRJ''S',     /,11X,'(  YR(J) = 0 IF THE J-TH V
     *ARIABLE IS IN THE BASIS.',/,11X,'IT CAN BE ZERO OTHERWISE TOO.)')
      DO 1037 J=1,N
1037  WRITE(6,1038) J,YR(J)
1038  FORMAT(1H ,15X,'YR(',I3,') = ',E15.5)
 2340 CONTINUE
C
C        IS THE IRMAX-TH COMPONENT OF Y NON-NEGATIVE FOR ALL OF THE
C        NON-BASIC VARIABLES?
C
      NUMBER = 0
      DO 27 L=1,N
27    IF(YR(L) .GE. TOLR2)    NUMBER=NUMBER+1
C
C     IF NUMBER = N, THEN THE IRMAX-TH COMPONENT OF Y IS NON-NEGATIVE
C        FOR ALL OF THE NON-BASIC VARIABLES.
C        HENCE THE PROBLEM HAS NO FEASIBLE SOLUTION.
C
      IF(NUMBER .EQ. N) GO TO 403
C
C        NO INDICATION OF PROBLEM INFEASIBILITY HAS BEEN FOUND THUSFAR
C        THE VARIABLE TO ENTER THE BASIS IS NOW DETERMINED
C
```

```
      IF (IWRIT2.EQ.1) WRITE(6,1045)
 1045 FORMAT(1H0,10X,'THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ''S
     *')
      IRMAX=0
      THETA=-1.D 25
      DO 1004 J=1,N
      IF(ISTAT(J) .EQ. 1) GO TO 1004
      IF(YR(J) .GE. TOLR2)   GO TO 1004
      W=-REDCOS(J)/YR(J)
      IF (IWRIT2.EQ.1) WRITE(6,1048) J,J,W
 1048 FORMAT(1H ,15X,'THE RATIO OF THE ',I3,'-TH NET PRICE TO YR(',I3,')
     * = ',E15.5)
      WW=W-THETA
      IF(WW .LT. 0.D0) GO TO 1004
      THETA=W
      IRMAX=J
 1004 CONTINUE
C
C        IRMAX = THE INDEX OF THE VARIABLE TO ENTER THE BASIS
C
      IF (IWRIT2.EQ.1) WRITE(6,1005) IL,IRMAX
 1005 FORMAT(1H0,10X,'THE ',I3,'-TH VARIABLE IS LEAVING THE BASIS.',/,11
     *X,'THE ',I3,'-TH VARIABLE IS ENTERING THE BASIS.')
C
C        UPDATE THE BASIS INVERSE: B1INV
C
C
C        COMPUTE THE Y COEFFICIENTS FOR THE INCOMING VARIABLE
C
      DO 1025 I=1,MP1
      Y1(I)=0.D0
      DO 1025 L=1,MP1
 1025 Y1(I)=Y1(I)+B1INV(I,L)*A1(L,IRMAX)
C
C        REPLACE THE IRMIN-TH COLUMN OF THE BASIS BY THE IRMAX-TH
C        COLUMN OF A
C
      IRMIN1=IRMIN
      DO 33 J=2,MP1
      WW=B1INV(IRMIN1,J)/Y1(IRMIN1)
      DO 37 L=1,MP1
   37 B1INV(L,J)=B1INV(L,J)-WW*Y1(L)
   33 B1INV(IRMIN1,J)=WW
C
C        UPDATE THE BASIC VARIABLES: INBASE, XB1, AND ISTAT
C
C        RECALL IL=IRMIN-1
C
      II=INBASE(IL)
      ISTAT(II)=0
      ISTAT(IRMAX)=1
      INBASE(IL)=IRMAX
      W=XB1(IRMIN1)/Y1(IRMIN1)
      DO 38 I=1,MP1
   38 XB1(I)=XB1(I)-Y1(I)*W
      XB1(IRMIN1)=W
      IF (IWRIT2.EQ.1) WRITE(6,541)
  541 FORMAT(1H0,'             THE BASIC VARIABLES ARE NOW')
      IF (IWRIT2.EQ.1) WRITE(6,542) (INBASE(K),K=1,M)
  542 FORMAT(1H ,15X,30(I3,', '))
      IF (IWRIT2.EQ.1) WRITE(6,543)
  543 FORMAT(1H0,10X,'THE VALUES OF THE BASIC VARIABLES ARE NOW')
      IF (IWRIT2.EQ.0) GO TO 2350
      DO 544 K=1,M
      KK=K+1
  544 WRITE(6,545) K,XB1(KK)
  545 FORMAT(1H ,15X,'XB(',I3,') = ',E15.5)
      WRITE(6,852) XB1(1)
  852 FORMAT(1H0,10X,'THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = ',E1
     *5.5)
 2350 CONTINUE
      GO TO 350
  401 IF(IWRIT1.EQ.1) WRITE(6,520)
  520 FORMAT(1H0,5X,'THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OP
     *TIMAL.')
      IF(IWRIT1.EQ.1) WRITE(6,521)
  521 FORMAT(1H ,5Y,'THE NONZERO VARIABLES ARE AS FOLLOWS:')
      IF(IWRIT1.EQ.0) GO TO 2360
      DO 522 LM=1,M
      LMM=LM+1
      K=INBASE(LM)
  522 WRITE(6,523)  K,XB1(LMM)
  523 FORMAT(1H ,15X,'X(',I3,') = ',E15.5)
 2360 CONTINUE
      IF(IWRIT1.EQ.1) WRITE(6,851) XB1(1)
  851 FORMAT(1H0,'     THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS ',
     *E15.5)
      IF(IWRIT1.EQ.1) WRITE(6,850)
  850 FORMAT(1H1)
      IF (ITERA.EQ.0) GO TO 2020
C
C        FORM THE SOLUTION TO DELTA BETA FOR THIS SAMPLE'S ESTAR
C
      DO 2599 J=1,N
 2599 DBX(J)=0.D0
      DO 2600 J=1,M
      L=INBASE(J)
 2600 DBX(L) = XB1(J+1)
      IF(IP1P2.EQ.1) GO TO 2602
      DO 2603 L=1,IP
      DB2(L)=DBX(L)
      DB1(L)=DBX(L+IP)
 2603 DELB(L,ITERA)=DB1(L)-DB2(L)+DBETA0(L)
      GO TO 2605
 2602 DO 2601 L=1,IP
      DB1(L) = DBX(L)
      DB2(L) = DBX(L+IP)
 2601 DELB(L,ITERA) = DB1(L) - DB2(L) + DBETA0(L)
 2605 CONTINUE
      DO 2609 L=1,IP
 2609 DELTA(L)=DELB(L,ITERA)
      IF (ITERA.GT.0) SUMOBJ=SUMOBJ - XB1(1)
      GO TO 2030
 2020 INTP12=IP1P2
      OPTOBJ=DABS(XB1(1))
      DO 2019 J=1,N
 2019 XBINT(J)=0.D0
      DO 2021 L=1,M
      K=INBASE(L)
 2021 XBINT(K)=XB1(L+1)
```

```
      SUMRES=0.D0
      DO 2022 NM=1,NOBS
      LNM=NM+2*IP
 2022 SUMRES=SUMRES+DABS(XBINT(LNM))
 2070 IF (ITERA.LE.ISAM) GO TO 200
403   WRITE(6,530)
530   FORMAT(1H0,5X,'ALL OF THE YRJS ARE NON-NEGATIVE.',/,11X,'HENCE THE
     * PROBLEM HAS NO FEASIBLE SOLUTION.')
      IF(IWRIT2.EQ.1) WRITE(6,850)
      GO TO 999
402   WRITE(6,1010)
1010  FORMAT(1H0,5X,'ALL OF THE REDUCED COSTS ARE NOT NON-POSITIVE.',/,1
     *1X,'HENCE THE DUAL SIMPLEX ALGORITHM IS NOT APPLICABLE.')
      WRITE(6,850)
      GO TO 999
200   CONTINUE
C
C          TEST FOR COMPLETION OF ALL SAMPLES
C
      IF (ITERA.EQ.ISAM) GO TO 2999
C
C          INCREMENT ITERA (THE SAMPLE INDEX COUNTER) BY 1.
C
      ITERA=ITERA+1
C
C          CALL SUBROUTINE NORMAL OR DOUBLE OR UNIFRM TO GENERATE A SET
C               OF RANDOM VARIABLES, EPSILONS, DISTRIBUTED AS NORMAL OR
C               DOUBLE EXPONENTIAL OF UNIFORM, RESPECTIVELY, WITH MEAN
C               ZERO AND VARIANCE ONE.
C
      IF(IOPTN.EQ.1) CALL NORMAL(NOBS,ESTAR)
      IF(IOPTN.EQ.2) CALL DOUBLE(NOBS,ESTAR)
      IF(IOPTN.EQ.3) CALL UNIFRM(NOBS,ESTAR)
      IF(IWRIT3.EQ.1) WRITE(6,3000) ITERA
 3000 FORMAT(1H0,5X,'SUPPLEMENTAL INFORMATION FROM THE MINI-MONTE CARLO'
     *,' STUDY',/,/,11X,'THE GENERATED N(0,1) EPSILONS',
     *5X,'SAMPLE NUMBER = ',I5)
      IF (IWRIT3.EQ.0) GO TO 3333
      DO 3002 K=1,NOBS
 3002 WRITE (6,3001) ESTAR(K)
 3001 FORMAT(1H ,15X,F12.5)
 3333 CONTINUE
C
C          CALL SUBROUTINE CONST TO CONSTRUCT XB1 FOR THIS SAMPLE
C
      IWRIT1=0
      IWRIT2=0
      IF(IWRIT3.EQ.1) IWRIT1=1
      IF(IWRIT4.EQ.1) IWRIT2=1
      CALL CONST(ESTAR,X,IP,NOBS,XB1,DBETA0,B1INV,A1)
      GO TO 300
 2999 CONTINUE
C
C          ALL SAMPLES COMPLETE
C
C
C          FORMS THE L1 ESTIMATE.
C
      IF(INTP12.EQ.1) GO TO 2700
      DO 2702 L=1,IP
      CAPB2(L)=XBINT(L)
      CAPB1(L)=XBINT(L+IP)
 2702 BHAT(L)=CAPB1(L)-CAPB2(L)+BETA0(L)
      GO TO 2701
 2700 DO 2040 L=1,IP
      CAPB1(L) = XBINT(L)
      CAPB2(L) = XBINT(L+IP)
 2040 BHAT(L) = CAPB1(L)-CAPB2(L)+BETA0(L)
 2701 CONTINUE
      IF(IWRIT1.EQ.1.OR.IWRIT2.EQ.1.OR.IWRIT3.EQ.1.OR.IWRIT4.EQ.1.OR.
     *IWRIT5.EQ.1.OR.IWRIT6.EQ.1.) WRITE(6,858)
  858 FORMAT(1H1)
      WRITE(6,2326)
 2326 FORMAT(1H0,5X,'THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0,',
     *' OF THE REGRESSION PARAMETER VECTOR, BETA')
      DO 3007 I=1,IP
 3007 WRITE(6,3008) I,BETA0(I)
 3008 FORMAT(1H ,12X,'LEAST SQUARES ESTIMATE OF BETA(',I3,') = ',F14.6)
      WRITE(6,3009)
 3009 FORMAT(1H0,5X,'MRS. A',1H','S ANSWER :',5X,
     *'THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH ',
     *'MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS: ')
      DO 3010 I=1,IP
 3010 WRITE(6,3011) I,BHAT(I)
 3011 FORMAT(1H ,12X,'L1 ESTIMATE OF BETA(',I3,') = ',F14.6)
C
C          PRINTOUT OF RESIIDUALS, R(I), I=1,NOBS
C
      WRITE(6,3019)
 3019 FORMAT(1H0,5X,'THE RESIDUALS, R(I),I=1,NOBS')
      DO 2067 L =1,NOBS
      LL=L+2*IP
 2067 WRITE(6,3018) XBINT(LL)
 3018 FORMAT(1H0,15X,F14.6)
      WRITE(6,3118) SUMRES
 3118 FORMAT(1H0,5X,'THE SUM OF THE ABSOLUTE RESIDUALS = ',F14.6)
      RESMAX=0.D0
      DO 3119 L=1,NOBS
      LL=L+2*IP
      IF(RESMAX.LE.DABS(XBINT(LL))) RESMAX=DABS(XBINT(LL))
 3119 CONTINUE
      WRITE(6,3120) RESMAX
 3120 FORMAT(1H0,5X,'THE MAXIMUM ABSOLUTE RESIDUAL = ',F14.6)
C
C          FORM THE VARIANCE OF DELTA BETA STAR
C          THESE CALCULATIONS USE THE EPSILON STARS.
C
      SAM = ISAM
      P = IP
      DO 2062 J=1,IP
      SMM=0.D0
      DO 2064 L=1,ISAM
 2064 SMM = SMM + DELB(J,L)
 2062 SUM(J)=SMM
      DO 2061 I=1,IP
      DO 2061 J=1,IP
      SUMSQ=0.D0
      SQSUM=0.D0
      SQSUM = SQSUM + SUM(J)*SUM(I)
      DO 2060 L=1,ISAM
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      SUMSQ = SUMSQ + DELB(J,L)*DELB(I,L)
 2060 CONTINUE
      VARDBS(I,J) = (SUMSQ-SQSUM/SAM)/(SAM-P)
 2061 CONTINUE
      IF(IWRIT6.EQ.1) WRITE(6,3003)
 3003 FORMAT(1H1,5X,'AUXILIARY RESULTS OF THE MINI-MONTE CARLO STUDY',/
     *,/,9X,'VALUES OF DELTA BETA STAR')
      IF(IWRIT6.EQ.0) GO TO 2380
      DO 3004 I=1,ISAM
 3004 WRITE(6,3005) I,(DELB(L,I),L=1,IP)
 3005 FORMAT(1H ,9X,'  SAMPLE NUMBER = ',I3,3X,10F11.5)
      WRITE(6,3012)
 3012 FORMAT(1H0,8X,'ESTIMATED COVARIANCE OF DELTA BETA STAR')
      DO 3013 I=1,IP
 3013 WRITE(6,3014) (VARDBS(I,J),J=1,IP)
 3014 FORMAT(1H ,9X,8F14.6)
 2380 CONTINUE
C
C         CALCULATION FOR SIGMA ESTIMATE
C
      SIGHAT=OPTOBJ/(SUMOBJ/SAM)
      IF(IWRIT6.EQ.1) WRITE(6,3020) SUMOBJ
 3020 FORMAT(1H0,'         SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER ',
     *'ALL SAMPLES = ',F14.6)
C
C         ESTIMATION OF SIGMA 5 AND OPTIIONAL PRINTOUT WHEN IWRIT6=1.
C
      SN=NOBS
      IF (IOPTN.EQ.1) CCC=2.506628275D0/2.D0
      IF(IOPTN.EQ.2) CCC=DSQRT(2.D0)
      IF(IOPTN.EQ.3) CCC=1.154700538D0
      SIGMA5=OPTOBJ*CCC/(SN-P)
      IF (IWRIT6.EQ.1) WRITE(6,3022) SIGMA5
 3022 FORMAT(1H0,/,1H ,5X,'AUXILIARY RESULT:  SIGMA HAT 5 = ',F14.6)
C
C         PRINTOUT OF ESTIMATE OF SIGMA, SIGMA HAT 4
C
      WRITE(6,2327)
 2327 FORMAT(1H1,5X,'MAIN RESULTS OF THE MINI-MONTE CARLO STUDY')
      WRITE(6,3023) SIGHAT
 3023 FORMAT(1H0,5X,'ESTIMATED VALUE OF SIGMA (SIGMA HAT 4) = ',F14.6)
C
C         CALCULATION THE VARIANCE OF DELTA BETA (=BETA)
C         BASED ON EPSILONS WITH VARIANCE OF SIGMA-SQUARED
C
      DO 2069 I=1,IP
      DO 2069 J=1,IP
 2069 VARB(I,J) =(SIGHAT**2)*VARDBS(I,J)
      WRITE(6,3015)
 3015 FORMAT(1H0,5X,'ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER',
     *' VECTOR (BETA) USING THIS ESTIMATE OF SIGMA')
      DO 3016 I=1,IP
 3016 WRITE(6,3017) (VARB(I,J),J=1,IP)
 3017 FORMAT(1H ,8X,8F14.6)
      WRITE(6,850)
  999 CONTINUE
      STOP
      END
      SUBROUTINE INVERT(A,N)
C
C         THIS SUBROUTINE INVERTS AN N BY N MATRIX
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(41,41)
   39 DO 51 I=1,N
      IF(A(I,I).EQ.0.D0) A(I,I)=0.1D-30
      A(I,I)=1.0D0/A(I,I)
      DO 52 J=1,N
      IF(J-I) 53,52,53
   53 A(I,J)=A(I,J)*A(I,I)
   52 CONTINUE
      DO 51 J=1,N
      IF(J-I) 54,51,54
   54 DO 56 K=1,N
      IF(K-I) 55,56,55
   55 A(J,K)=A(J,K)-A(J,I)*A(I,K)
   56 CONTINUE
      A(J,I)=-A(J,I)*A(I,I)
   51 CONTINUE
      RETURN
      END
      SUBROUTINE CONST(OBS,X,IP,NOBS,XB1,BETA0,B1INV,A1)
C
C         THIS IS SUBROUTINE CONSTRUCT.  THIS SUBROUTINE:
C
C         1. RANDOMLY SELECTS BETWEEN LINEAR PROBLEMS
C         P1 AND P2 EACH WITH A PROBABILITY OF ONE-HALF.
C
C         2. COMPUTES THE VALUE OF BETA0, THE LEAST
C         SQUARES ESTIMATOR WHICH SERVES AS THE INITIAL SYMMETRIC
C         ESTIMATOR.
C
C         3. COMPUTES THE INITIAL BASIC VARIABLES,
C         XB1, FOR THE DUAL SIMPLEX ALGORITHM.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION OBS(20),X(20,10),XB1(41),BETA0(10),B1INV(41,41)
      DIMENSION A1(41,80),YMXB(20),XTY(10)
      COMMON/XTXIM/XTX(41,41)
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      COMMON/NFEED/NSEED
      COMMON/ICONST/ITERA,ITIP,MP1,IP1P2
C
C         DIMENSIONS OF ABOVE ARRAYS ARE. . .
C              OBS(NOBS), X(NOBS,IP),XB1(M+1),BETA0(IP),B1INV(M+1,M+1)
C              A1(M+1,N),YMXB(NOBS),XTX(IP,IP),XTY(IP)
C
C         THE RANDOM SELECTION OF PROBLEMS P1 AND P2, EACH WITH
C         PROBABILITY OF ONE-HALF USING SUBROUTINE RAND.
C
C         IF IP1P2 = 1 THEN P1 IS USED,
C         IF IP1P2 = 2 THEN P2 IS USED.
C
      NSEED=NSEED + ITERA*70968
      CALL RAND(U)
      IP1P2=1
      IF(U.GT.5.D-1) IP1P2=2
      IF(IWRIT1.EQ.1) WRITE(6,50) IP1P2
   50 FORMAT(1H0,'         THE PROBLEM P',I1,' HAS BEEN SELECTED')
C
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C          CALCULATION OF THE LEAST SQUARES ESTIMATE OF BETA          CONS0040
C          USING SUBROUTINES XTXINV AND INVERT.                       CONS0041
C                                                                     CONS0042
      CALL XTXINV(X,IP,NOBS)                                          CONS0043
      DO 31 I=1,IP                                                    CONS0044
      SUM=0.D0                                                        CONS0045
      DO 30 J=1,NOBS                                                  CONS0046
   30 SUM = SUM + X(J,I)*OBS(J)                                       CONS0047
      XTY(I) = SUM                                                    CONS0048
   31 CONTINUE                                                        CONS0049
      DO 41 J=1,IP                                                    CONS0050
      SUM =0.D0                                                       CONS0051
      DO 40 I=1,IP                                                    CONS0052
      SUM = SUM + XTX(J,I)*XTY(I)                                     CONS0053
      BETA0(J) = SUM                                                  CONS0054
   40 CONTINUE                                                        CONS0055
   41 CONTINUE                                                        CONS0056
      IF(IWRIT1.EQ.1) WRITE(6,55)                                     CONS0057
   55 FORMAT(1H0,'          VALUE OF BETA0 TO COMPUTE RHS')           CONS0058
      IF(IWRIT1.EQ.1) WRITE(6,56) (BETA0(J),J=1,IP)                   CONS0059
   56 FORMAT(1H ,9X,8F14.6)                                           CONS0060
C                                                                     CONS0061
C          COMPUTATION OF THE INITIAL VALUES OF THE BASIC VARIABLES,  CONS0062
C          XB1, TO BE USED IN THE DUAL SIMPLEX ALGORITHM.             CONS0063
C                                                                     CONS0064
      DO 11 I=1,NOBS                                                  CONS0065
      SUM=OBS(I)                                                      CONS0066
      DO 12 J=1,IP                                                    CONS0067
   12 SUM = SUM - X(I,J)*BETA0(J)                                     CONS0068
   11 YMXB(I) = SUM                                                   CONS0069
C                                                                     CONS0070
C          YMXB ARE THE RHS OF THE CONSTRAINTS IN THE FORM Y-XB.      CONS0071
C                                                                     CONS0072
C          TEST TO DETERMINE IF PROBLEM P1 OR P2 HAS BEEN             CONS0073
C          SELECTED AND ADJUST YMXB ACCORDINGLY.                      CONS0074
C                                                                     CONS0075
      IF (IP1P2.EQ.2) GO TO 15                                        CONS0076
      DO 16 I=1,NOBS                                                  CONS0077
   16 YMXB(I) = -YMXB(I)                                              CONS0078
   15 CONTINUE                                                        CONS0079
      IF(IWRIT1.EQ.0) GO TO 2392                                      CONS0080
      WRITE(6,60)                                                     CONS0081
   60 FORMAT(1H0,'          THE RHS, YMXB, FOR P1 OR P2')             CONS0082
      DO 61 J=1,NOBS                                                  CONS0083
   61 WRITE(6,62) YMXB(J)                                             CONS0084
   62 FORMAT(1H ,10X,F14.6)                                           CONS0085
 2392 CONTINUE                                                        CONS0086
C                                                                     CONS0087
C          CALCULATION OF XB1                                         CONS0088
C                                                                     CONS0089
      DO 13 I=1,MP1                                                   CONS0090
      SUM=0.D0                                                        CONS0091
      DO 14 J=1,NOBS                                                  CONS0092
   14 SUM = SUM + (B1INV(I,J+1) - B1INV(I,1+NOBS+J))*YMXB(J)          CONS0093
   13 XB1(I) = SUM                                                    CONS0094
      RETURN                                                          CONS0095
      END                                                             CONS0096
      SUBROUTINE XTXINV(X,IP,NOBS)                                    XTXI0001
C                                                                     XTXI0002
C                                                                     XTXI0003
C          SUBROUTINE XTXINV . . .                                    XTXI0004
C           CALCULATES X'X INVERSE FOR USE IN FORMING BETA0.          XTXI0005
C                                                                     XTXI0006
C                                                                     XTXI0007
      IMPLICIT REAL*8 (A-H,O-Z)                                       XTXI0008
      DIMENSION X(20,10)                                              XTXI0009
      COMMON/XTXIM/XTX(41,41)                                         XTXI0010
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6         XTXI0011
      DO 22 I=1,IP                                                    XTXI0012
      DO 21 K=1,IP                                                    XTXI0013
      SUM=0.D0                                                        XTXI0014
      DO 20 J=1,NOBS                                                  XTXI0015
      SUM = SUM + X(J,I)*X(J,K)                                       XTXI0016
   20 CONTINUE                                                        XTXI0017
      XTX(I,K)=SUM                                                    XTXI0018
   21 CONTINUE                                                        XTXI0019
   22 CONTINUE                                                        XTXI0020
      CALL INVERT(XTX,IP)                                             XTXI0021
      IF (IWRIT1.EQ.0) GO TO 2395                                     XTXI0022
      WRITE(6,67)                                                     XTXI0023
   67 FORMAT(1H0,'          THE INVERSE OF XTX')                      XTXI0024
      DO 68 I=1,IP                                                    XTXI0025
   68 WRITE(6,69) (XTX(I,K),K=1,IP)                                   XTXI0026
   69 FORMAT(1H ,9X,8F14.6)                                           XTXI0027
 2395 CONTINUE                                                        XTXI0028
C          NOW XTX IS REALLY XTX INVERSE.                             XTXI0029
      RETURN                                                          XTXI0030
      END                                                             XTXI0031
      SUBROUTINE RAND(U)                                              RAND0001
C                                                                     RAND0002
C                                                                     RAND0003
C          SUBROUTINE RAND. . .                                       RAND0004
C           GENERATES UNIFORM (0,1) RANDOM NUMBERS.                   RAND0005
C           NSEED IS A RANDOM TEN DIGIT INTEGER SUPPLIED IN MAIN.     RAND0006
C                                                                     RAND0007
C                                                                     RAND0008
      IMPLICIT REAL*8 (A-H,O-Z)                                       RAND0009
      COMMON/NFEED/NSEED                                              RAND0010
      MD=2147483647                                                   RAND0011
      ML=764261123                                                    RAND0012
      FMD=DFLOAT(MD)                                                  RAND0013
      NSEED=NSEED*ML                                                  RAND0014
      IF (NSEED.LT.0) NSEED=NSEED+MD                                  RAND0015
      U=DFLOAT(NSEED)                                                 RAND0016
      U=U/FMD                                                         RAND0017
      RETURN                                                          RAND0018
      END                                                             RAND0019
      SUBROUTINE NORMAL(NOBS,OBS)                                     NORM0001
C                                                                     NORM0002
C                                                                     NORM0003
C          SUBROUTINE NORMAL. . .                                     NORM0004
C           GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS       NORM0005
C           NORMAL (0,1)) USING A RANDOM NORMAL GENERATOR             NORM0006
C           CALLED BUTLER'S ALGORITHM AS ADAPTED FROM RAND.           NORM0007
C                                                                     NORM0008
C                                                                     NORM0009
      DIMENSION RANDM(20)                                             NORM0010
      DIMENSION C(6),X(257),U(3),STORE(256),R(256)                    NORM0011
      COMMON/NFEED/NSEED                                              NORM0012
      DATA C/2.515517,.802853,.010328,1.43279,.189269,.0013081/       NORM0013
```

```
      DATA IX,JX,STORE /Z7FFFDEC3,Z7DBD1557,256*0.0/                    NORM0014
      DOUBLE PRECISION OBS(20),UNIF                                     NORM0015
      CONST = SQRT(1./(2.*3.14159))                                     NORM0016
      X(1) =-3.6                                                        NORM0017
      X(257)=3.6                                                        NORM0018
      FOLD=0.0                                                          NORM0019
      RAT=1./256.                                                       NORM0020
      RAN=0.0                                                           NORM0021
      DO 10 I=1,255                                                     NORM0022
      RAN=RAN+RAT                                                       NORM0023
      IF (I.GT.128) GO TO 12                                            NORM0024
      T = SQRT(-2.*ALOG(RAN))                                           NORM0025
      GO TO 14                                                          NORM0026
   12 T = SQRT(-2.*ALOG(1.-RAN))                                        NORM0027
   14 Z = T - (C(1)+C(2)*T+C(3)*T**2)/(1.+C(4)*T+C(5)*T**2+C(6)*T**3)   NORM0028
      IF (I.LT.129) Z=-Z                                                NORM0029
      X(I+1)=Z                                                          NORM0030
      FNEW=CONST*EXP(-Z**2/2.)                                          NORM0031
   20 R(I) = (FNEW-FOLD)/(FNEW+FOLD)                                    NORM0032
   10 FOLD=FNEW                                                         NORM0033
      FNEW=0.0                                                          NORM0034
      R(256) = (FNEW-FOLD)/(FNEW+FOLD)                                  NORM0035
      SUM=0.0                                                           NORM0036
      SS = 0.0                                                          NORM0037
      DO 30 L=1,NOBS                                                    NORM0038
      IX=IX*65539                                                       NORM0039
      JX=JX*262147                                                      NORM0040
      RANDM(L)=.4656613E-9*FLOAT(IABS(IX+JX))                           NORM0041
      I=256.*RANDM(L)+1.0                                               NORM0042
      DO 32 K=1,3                                                       NORM0043
      CALL RAND(UNIF)                                                   NORM0044
   32 U(K)=UNIF                                                         NORM0045
      Z=X(I+1)-X(I)                                                     NORM0046
      IF (U(3).GT.ABS(R(I))) GO TO 34                                   NORM0047
      RANDM(L)=X(I)+Z*U(1)                                              NORM0048
      GO TO 36                                                          NORM0049
   34 IF (R(I).LT.0.0) GO TO 50                                         NORM0050
      RANDM(L)=AMAX1(U(1),U(2))                                         NORM0051
      GO TO 52                                                          NORM0052
   50 RANDM(L)=AMIN1(U(1),U(2))                                         NORM0053
   52 RANDM(L)=X(I)+Z*RANDM(L)                                          NORM0054
   36 CONTINUE                                                          NORM0055
      OBS(L)=RANDM(L)                                                   NORM0056
   30 CONTINUE                                                          NORM0057
      RETURN                                                            NORM0058
      END                                                               NORM0059
      SUBROUTINE DOUBLE(NOBS,DBLEXP)                                    DOUB0001
C                                                                       DOUB0002
C          SUBROUTINE DOUBLE . . .                                      DOUB0003
C            GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS        DOUB0004
C            DOUBLE EXPONENTIAL VARIABLES WITH MEAN ZERO AND            DOUB0005
C            VARIANCE ONE.)                                             DOUB0006
C                                                                       DOUB0007
      IMPLICIT REAL*8 (A-H,O-Z)                                         DOUB0008
      DIMENSION DBLEXP(20),UXX(2)                                       DOUB0009
      DO 10 I=1,NOBS                                                    DOUB0010
      DO 20 J=1,2                                                       DOUB0011
      CALL RAND(UX)                                                     DOUB0012
   20 UXX(J)=UX                                                         DOUB0013
      UW=(1.0D0/DSQRT(2.D0))*(DLOG(UXX(2))-DLOG(UXX(1)))                DOUB0014
   10 DBLEXP(I)=UW                                                      DOUB0015
      RETURN                                                            DOUB0016
      END                                                               DOUB0017
      SUBROUTINE UNIFRM(NOBS,UVAR1)                                     UNIF0001
C                                                                       UNIF0002
C          SUBROUTINE UNIFRM . . .                                      UNIF0003
C            GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS        UNIF0004
C            UNIFORM VARIABLES WITH MEAN ZERO, VARIANCE ONE.)           UNIF0005
C                                                                       UNIF0006
      IMPLICIT REAL*8 (A-H,O-Z)                                         UNIF0007
      DIMENSION UVAR1(20)                                               UNIF0008
      DO 10 I=1,NOBS                                                    UNIF0009
      CALL RAND(UZO)                                                    UNIF0010
   10 UVAR1(I)=UZO*DSQRT(12.D0)                                         UNIF0011
      RETURN                                                            UNIF0012
      END                                                               UNIF0013
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The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program

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20. Cont.

for obtaining unbiased L_1 estimators and estimates of their covariances. These estimated covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 65 provides an analogous treatment of L_1 estimation subject to linear constraints on the parameters.

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